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OPTIMAL INVESTMENT ALLOCATION IN DECENTRALIZED MARKETS

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Abstract

This paper makes three contributions to the Feldman-Mahalanobis (F-M) model. First, it overcomes the limitation of the original model, which assumes a passive role of consumption demand, by extending the F-M model through the introduction of intertemporal maximization of consumption. Second, it shows that decentralized markets can mimic the dynamic behavior of the centrally planned economy with two sectors, consumption and investment goods. This is accomplished by using Cobb-Douglas production functions in both sectors. Third, in contrast with the F-M model in which the solutions are unstable, this paper proves the stability of the steady state solutions.

Keywords: Investment allocation; Two sector models; Dynamic optimization.

1 INTRODUCTION

Feldman's (1928) two-sector growth model is widely used as a benchmark to study the effects of the investment allocation on economic growth. This is a model with a consumption and an investment sector in which capital goods can be used to increase the capacity of either sectors. At any given instant of time the productive capacity of each sector is quasi-fixed and non-shiftable (that is, the technology can be describe by Leontief production functions), but over time the proportions can be continuously altered by allocating new investments to the one or the other of the two sectors. One of its main characteristics is that the economy is relatively closed (in the sense it has limited access to foreign savings, capital and technology) and the limiting factor of production is the stock of capital goods. This model of planned development is an approach of accelerated accumulation in which capital goods feeds upon itself and consumption is temporarily compressed.

This model was originally developed in the early days of Soviet planning where the question of the distribution of capital goods when such goods are in short supply. According to Chowdhury and Kirkpatrick (1994, p. 24-25), "[t]he model was later modified by Domar (1957), and Mahalanobis (1953) developed a very similar model where production of 'machines to produce machines' is the central issues". Dutt (1990, p. 120) considers that no discussion related to models with investment and consumption good sectors is complete without considering the contribution of Feldman (1928), and Dobb (1960) pointed out its importance as well.

But as noted by Amann (2000, p. 4)

> [...] [a]lthough the planning approaches typified by the F-M model was influential in the formulation of development strategies in centrally planned economies, they have increasingly fallen out of favour as development economists have placed a greater and greater emphasis on decentralization and the market in the process of resource allocation. Despite the demise of the planning models, the capital goods sector has continued receive attention within the debate on development and industrialization.

Arguably one of the limitations of the original F-M model is that it does not take into account the role of demand on decisions of investment allocation in decentralized markets. Domar (1957, p. 254) wrote on this point that

> Feldman's task was to explain to the Soviet planners the basic principles of economic growth and to furnish them with several alternative patterns of development, depending on the magnitudes of the rate of investment allocation and of the capital coefficients. It was up to the planners to choose the optimum path, depending on their own objective, and on their evaluation of existing economic and political conditions and possibilities. Such an evaluation of "the state of the mind of the masses" was in a sense a search for a discount function, but what exactly would be gained by an attempt to formalize it?

In order to mitigate the limitations of the F-M model in relation to the passive role of per capita consumption demand[1], Bose (1968) put Feldman's model in an intertemporal dynamic model a la Ramsey (1928). Meanwhile, Weitzman (1971) extended it by including a third sector of intermediate goods and services used indirectly in producing both consumption and investment goods. But their analyses were carried out assuming a central planner along the lines of the original models.

1 Araujo and Teixeira (2002) have shown that an alternative route to eliminate the limitations of the F-M model in relation to the passive role of consumption demand it to treat it as a particular case of Pasinetti's (1981) model of structural change.

Here we purport to develop an analysis of investment allocation in the lines suggested by Feldman and Mahalanobis but ascribing full attention to the role of decentralized markets. It is known that in general an optimal growth problem has a decentralized interpretation in terms of the market behavior of a representative agent (see ARAUJO, 2004). In this vein we extend the intertemporal dynamic F-M model to the case of a decentralized economy. The objective is to show that this model is consistent with optimal investment allocation in a decentralized competitive equilibrium. However, the duality shown in this paper does not hold squarely but it is accomplished by using Cobb-Douglas production functions in both sectors since it does not hold for the original Leontief production functions used in Feldman-Mahalanobis model due to the labor market failure. By following this approach we reemphasize the relevance of the analysis of investment allocation to the quest of economic growth.

It is important to mention that there are other two-sector models that take into account the role of capital accumulation. In the Neo-classical tradition the work of Uzawa (1961) is distinctive. One of the main differences between our analysis and Uzawa's approach is that here we focus on allocation of flows of capital goods while he focused on allocation of stocks of capital goods. We acknowledge that in the long run the steady state equilibrium will lead to a coincidence between the allocation of flows and stocks. But here we are also able to establish the optimal rate of investment allocation and show the equivalence between the decisions of investment allocation in decentralized markets and optimal command. A possible advantage of our approach is that the uniquely determined balanced growth equilibrium is found without the hypothesis that the consumption-goods sector is always more capital-intensive than the investment-goods sector, necessary to the Uzawa model.

We also provide a complete characterization of the convergence and stability analysis of investment allocation in a decentralized economy. The optimal solutions of the model are proved to be stable, which is in sharp contrast with the F-M model, where the solutions are unstable. The paper is structured as follows. The next section develops the decentralized model and presents the stability analysis, showing that the steady state solutions are stable. Section 3 introduces investment specific technological progress in the model and presents the intertemporal consumption maximization. The concluding remarks appear in Section 4.

2 A DECENTRALIZED VERSION OF THE FELDMAN--MAHALANOBIS MODEL

The model considers two sectors: one produces a capital goods and is denoted by subscript 1 and the other is the corresponding consumption goods sector, denoted by 2. The capital goods are used by both sectors but once installed, they cannot be transferred from one sector to the other (non-shiftability assumption). A proportion λ of the current production of the investment sector is allocated to itself while the remaining, 1-λ, is allocated to sector 1 ($1 \geq \lambda \geq 0$). For the sake of convenience only, it is assumed that there is no depreciation of capital goods. The investment goods cannot be imported and the production of capital goods does not depend on the production of consumption goods sector. The technology in both sectors is described by Cobb--Douglas production functions.

$$X_1 = A_1 K_1^{\alpha} L_1^{1-\alpha} \ , A_1 > 0 \tag{1}$$

where X_1 stands for the production of physical capital, K_1 refers to the stock of physical capital in the investment sector and L_1 is the working force in sector 1.

$$X_2 = A_2 K_2^{\beta} L_2^{1-\beta} \ , \ A_2 > 0 \tag{2}$$

where X_2 refers to the production of consumption goods, K_2 is the stock of physical capital and L_2 stands for working force in the goods sector. Total working force (L) is employed in one of the two sectors according to a fixed share (a). So we have:

$$L_1 + L_2 = L \tag{3}$$

$$L_1 = aL \tag{4}$$

The population is assumed to increase at an exogenous rate *n*:

$$\frac{\dot{L}}{L} = n \Rightarrow L = e^{nt} \tag{5}$$

From equations (3)-(5) it follows that the working force in each of the sectors grows at the same rate *n* as well. Therefore at each point in time total population is known and given. We can normalize the variables by the total population, so as to write them in per-capita terms:

$$x_1 = A_1 k_1^{\alpha} a^{1-\alpha} \quad , \quad A_1 > 0 \tag{6}$$

$$x_2 = A_2 k_2^{\beta} (1-a)^{1-\beta} \quad , \quad A_2 > 0 \tag{7}$$

where the lower case letters denote the variables in per-capita units. The variation of the stock of capital in sector 1 depends only on the proportion of the total output of this sector that is allocated to itself:

$$\dot{K}_1 = \lambda X_1 - \delta K_1 \tag{8}$$

In per capita units this expression can be rewritten as:

$$\dot{k}_1 = \lambda A_1 k_1^{\alpha} a^{1-\alpha} - (n+\delta) k_1 \tag{9}$$

The variation of the capital stock in sector 2 is given by:

$$\dot{K}_2 = (1-\lambda) X_1 - \delta K_2 \tag{10}$$

This expression can be rewritten in per capita unit as:

$$\dot{k}_2 = (1-\lambda)A_1 k_1^{\alpha} a^{1-\alpha} - (n+\delta)k_2 \qquad (11)$$

Considering the system of differential equations formed by expressions (9) and (11), we can inquire about the stability of the model. Let us rewrite this system as:

$$\dot{k}_1(t) = f_1(t,k_1,k_2) = \lambda A_1 k_1^{\alpha} a^{1-\alpha} - (n+\delta)k_1 \qquad (9)'$$
$$\dot{k}_2(t) = f_2(t,k_1,k_2) = (1-\lambda)A_1 k_1^{\alpha} a^{1-\alpha} - (n+\delta)k_2 \qquad (11)'$$

The systems formed of equations (9)' and (11)' with initial conditions $[k_1(0),k_2(0)] = [k_1^0, k_2^0]$ is said to have a *solution* on the interval $I: \alpha < t < \beta$ if there exists a set of two functions $k_1(t) = \varphi_1(t)$ and $k_2(t) = \varphi_2(t)$ differentiable at all points in the interval *I* and that satisfy the system of equations (9)' and (11)' at all points in this interval. The functions f_1 and f_2 and their partial derivatives with respet to k_1 and k_2 are continuous in a region Ω of tk_1k_2 *txy*-space defined by $\alpha < t < \beta$, $\gamma_1 < k_1 < \gamma_2$, $\beta_1 < k_2 < \beta_2$ where: $\frac{\partial f_1}{\partial k_1} = \lambda A_1 \alpha k_1^{\alpha-1} a^{1-\alpha} - (n+\delta)$, $\frac{\partial f_1}{\partial k_2} = 0$, $\frac{\partial f_2}{\partial k_1} = (1-\lambda)A_1 k_1^{\alpha-1} a^{1-\alpha}$ and $\frac{\partial f_2}{\partial k_2} = -(n+\delta)$. From expressions (9)' and (11)' evaluated in steady state the value of the capital stock in sectors 1 and 2 may be reckoned as:

$$k_1^* = \left(\frac{\lambda A_1}{n+\delta}\right)^{\frac{1}{1-\alpha}} \qquad (12)$$

$$k_2{}^* = \frac{A_1(1-\lambda)\left(a^{\frac{1}{\alpha}}\lambda A_1\right)^{\frac{\alpha}{1-\alpha}}}{(n+\delta)^{\left(\frac{1}{1-\alpha}\right)}} \quad (13)$$

- *Proposition 1*: The equilibrium given by expressions (15) and (16) is is locally asymptotically stable.
- *Proof*: The Jacobian matrix associated with the linearized vector field is given by:

$$J = \begin{pmatrix} \lambda A_1 k_1^{\alpha-1} a^{1-\alpha} - (n+\delta) & 0 \\ (1-\lambda) A_1 k_1^{\alpha-1} a^{1-\alpha} & -(n+\delta) \end{pmatrix} \quad (14)$$

The eigenvalues of J are given by: $\delta_{1,2} = \frac{trJ}{2} \pm \frac{\sqrt{tr^2 J - 4\det J}}{2}$. Where:

$$\det J = -(n+\delta)[\lambda A_1 k_1^{\alpha-1} a^{1-\alpha} - (n+\delta)] \quad (15)$$

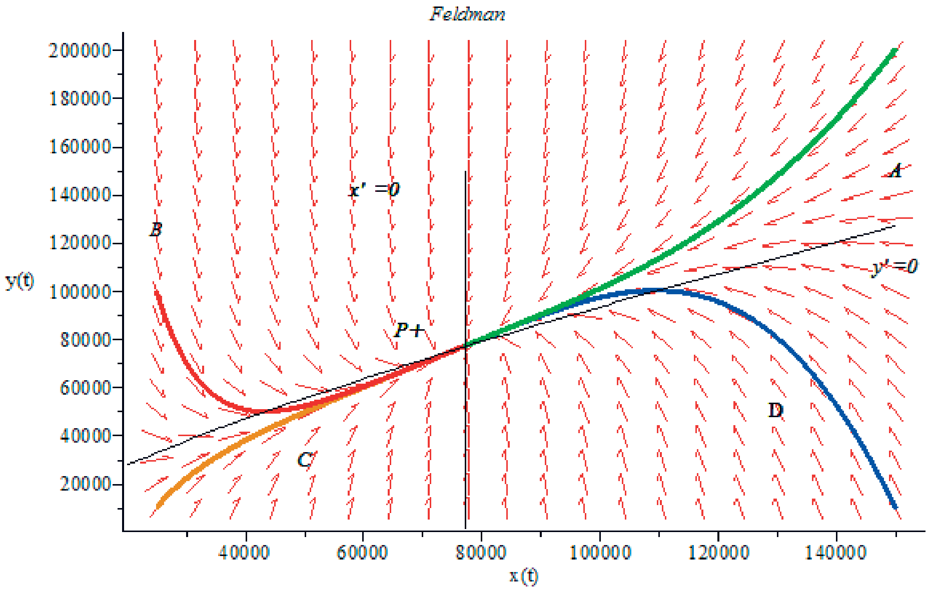
$$trJ = \lambda A_1 k_1^{\alpha-1} a^{1-\alpha} - 2(n+\delta) \quad (16)$$

Evaluating *det J* and *tr J* at $(k_1{}^*, k_2{}^*)$ yields $\det J = (n+\delta)^2(1-a^{1-\alpha}) > 0$ and $trJ = -(n+\delta) < 0$. Hence $\delta_1 < 0$ and $\delta_2 < 0$ which allows us to conclude that the equilibrium is locally asymptotically stable and are shown in Graph 1. q.e.d.

It is worth to mention that the stability of the model does not rely on the value of rate of investment allocation. It may take any value within the possible range, $0 \le \lambda \le 1$, and the value of *det J* and *tr J* remain unaltered. However, the value of this variable affects the steady state values of the stock of capital goods in sectors 1 and 2, as shown in Graph 1 below. By choosing, for instance $\lambda = 1/2$ and considering that $A_1 = 1$, $n = 0.03$ $\lambda^* = \frac{1}{3}$, $n = 0.03$ and $\alpha = 0.75$ the equilibrium point is reckoned as: $(k_1{}^*, k_2{}^*) =$ $(77170.49383, 77160.49383)$. Graph 1 shows this result.

Graph 1

The equilibrium point $(k_1^*, k_2^*) = (77170.49383, 77160.49383)$ is an asymptotically stable node, and direction field for the system (9)′ and (11)′. Here $\lambda = \frac{1}{2}$, $n = 0.03$ $\lambda^* = \frac{1}{3}$, $n = 0.03$ and $\alpha = 0.75$

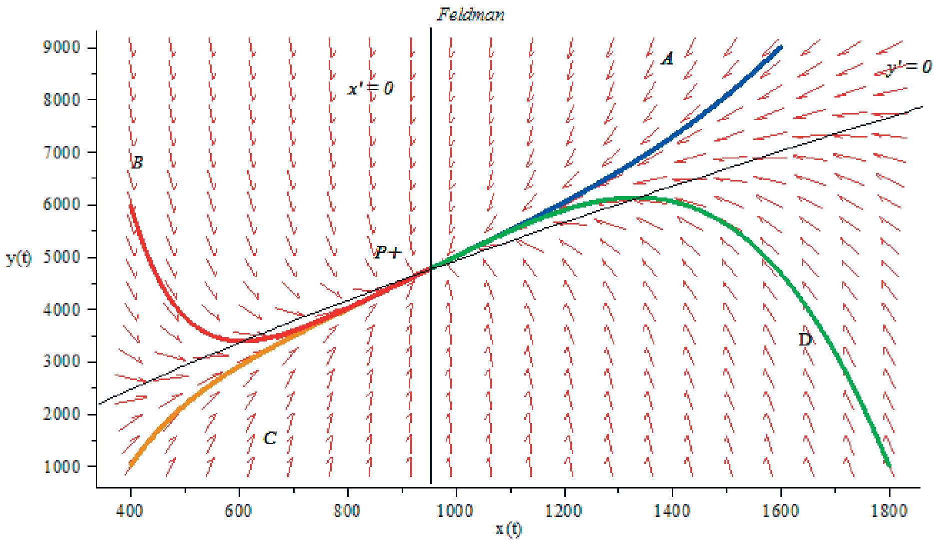


Source: Elaborated by the authors.

By considering another value for λ, let us say $\lambda = \frac{1}{6}$ and keeping the values for $n = 0.03$ $\lambda^* = \frac{1}{3}$, $n = 0.03$ and $\alpha = 0.75$ we obtain another equilibrium point, which is also stable.

Graph 2

The equilibrium point $(k_1^*, k_2^*) = (952.598692, 4762.993446)$ $P+ = (k_1^*, k_1^*) = (952.5986892, 4762.993446)$ is an asymptotically stable node, and direction field for the system (9)′ and (11)′. Here $\lambda = \frac{1}{6}$, $n = 0.03$ $\lambda^* = \frac{1}{3}$, $n = 0.03$ and $\alpha = 0.75$ λ = 0.166, n = 0.03

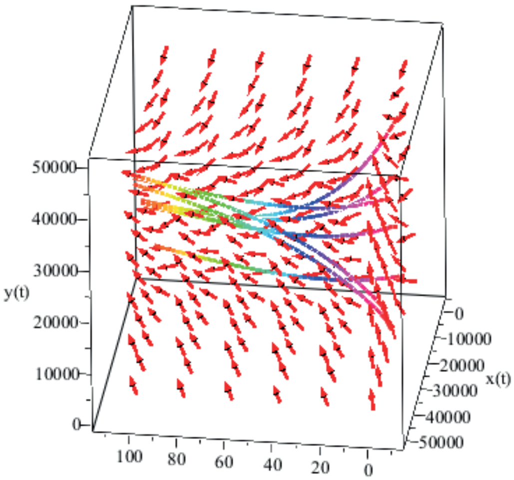


Source: Elaborated by the authors.

In order to better illustrate the dynamics of the model, let us consider that the central planner chooses a dynamic path for the rate of investment allocation. The aim is to show a three – dimension phase portrait of the dynamic system formed by expressions (9)', (11)' and a new function that conveys the dynamic path of the rate of investment allocation. Let us consider that the initial value of the rate of investment allocation is 1/6 and that the central planner intends to increase asymptotically this rate to ½ through an exponential function. A possible functional form for this expression is given by: $\lambda(t) = \frac{1}{2} - \frac{1}{3}\exp(-t)$. By considering $n = 0.03$ and $\alpha = 0.75$, the dynamical behavior of the dynamical system may be illustrated by the following graphs.

Graphic 3

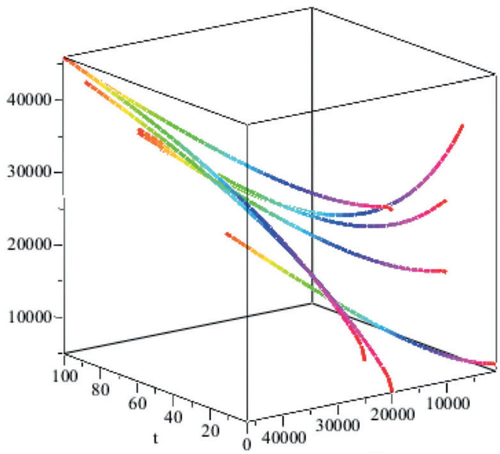
The equilibrium point



Source: Elaborated by the authors.

Graphic 4

The equilibrium point



Source: Elaborated by the authors.

Note that when the value of t increases the system departs from its initial equilibrium to reach asymptotically its final position. One shortcoming of this analysis is that it is not possible to determine the optimal rate of investment allocation. On one hand, a higher value for λ implies a smaller level of consumption in the short run once higher proportions of the production of the capital goods sector will be allocated to itself and smaller proportions of this production will be allocated to the consumption goods sector. On the other hand, the higher allocation of the production of capital goods that is being allocated to this sector will allow an expansion of the production of this sector and, as a consequence, the expansion of the stock of capital goods of the economy as a whole. This higher stock of capital goods will allow a higher production of consumption goods in the future. In face of this trade-off between current and future consumption, only a structure of dynamic optimization will allow us to choose the optimal rate of investment allocation to maximize intertemporal consumption. This analysis will be carried out in the next section.

3 INVESTMENT SPECIFIC TECHNOLOGICAL PROGRESS AND OPTIMAL INVESTMENT ALLOCATION

One of the possible shortcomings of the previous analysis is the fact that technical progress is not considered. Arguably, if larger portions of technical progress are embodied into capital goods then the decisions on investment allocation are also decisions on the allocation of technical progress. This viewpoint is supported by Collechia and Schreyer (2002) who have shown that not only the investment but also its allocation play an important role in harvesting the benefits of technological change (especially information and communication technologies) embodied in capital goods. While disembodied technological change affects output growth independently of capital accumulation, embodied technological change requires investment to do so.

This approach is due to Solow (1957) and was followed by a number of authors such as Phelps (1962) and Nelson (1964). It considers that each vintage of capital goods is the result of investment in period v and has a rate m of embodied technical progress and depreciates at a rate δ:

$$K_1(v,t) = l\ (v)I(v)e^{mv+d\,(v-t)} \tag{17}$$

The stock of capital goods in sector 1 in period *t* is given by the integral over the ages of different vintages of capital goods that are installed in this sector.

$$K_1(t) = \int_0^t K_1(v,t)dv = \int_0^t l\ (v)I(v)e^{m\ +d\,(v-t)}dv \tag{18}$$

Since $I(v) = X_1(v) = A_1\left[K_1(v)\right]^{\alpha}\left[L_1(v)\right]^{1-\alpha}$ Expression (18) may be rewritten as:

$$K_1(t) = \int_0^t K_1(v,t)dv = \int_0^t \lambda(v)A_1\left[K_1(v)\right]^{\alpha}\left[L_1(v)\right]^{1-\alpha}e^{mv+\delta(v-t)}dv \tag{19}$$

By differentiating both sides of this expression and applying the Fundamental Theorem of Calculus we conclude that the variation in the stock of capital goods in sector 1 is given by:

$$\dot{K}_1(t) = \lambda(t)A_1\left[K_1(t)\right]^{\alpha}\left[L_1(t)\right]^{1-\alpha}e^{mt} - \delta K_1(t) \tag{20}$$

Now let us consider that $A_1(t) = A_1e^{mt}$ captures the investment specific technical progress the above expression may be written as:

$$\dot{K}_1(t) = \lambda(t)A_1(t)\left[K_1(v)\right]^{\alpha}\left[L_1(v)\right]^{1-\alpha} - \delta K_1(t) \tag{21}$$

The law of motion for the stock of capital obtained from the Solow's specification is similar to the previous one with the technological parameter, A_1, constant but now $A_1(t)$ is an exponential function of time and conveys the Hicks neutral technical progress. By adopting the same procedure in relation to K_2 we have that:

$$\dot{K}_2(t) = \left[1 - \lambda(t)\right] A_1(t) \left[K_1(v)\right]^{\alpha} \left[L_1(v)\right]^{1-\alpha} - \delta K_2(t) \qquad (22)$$

In per capita terms the expressions (21) and (22) may be rewritten as:

$$\dot{k}_1 = \lambda A_1(t) k_1^{\alpha} a^{1-\alpha} - (n+\delta) k_1 \qquad (21)'$$

$$\dot{k}_2 = (1-\lambda) A_1(t) k_1^{\alpha} a^{1-\alpha} - (n+\delta) k_2 \qquad (22)'$$

One of the properties of this model is that in the short run the higher the rate of investment allocation the higher the growth rate of the capital goods sector and smaller the growth rate of the consumption goods sector. But in the long run a higher rate of investment allocation implies a higher growth rate of both sectors since the consumption goods sector feeds upon the capital goods one. Hence an optimal value of the rate of investment allocation maximizes the intertemporal production of consumption goods. In order to determine the optimal rate of capital accumulation let us assume that the central planner solves the following problem:

In order to determine the optimal rate of investment allocation, let us assume initially that a central planner wants to maximize the family welfare. The instantaneous utility function that is defined over consumption goods is assumed to be of constant elasticity of substitution: $U(x_2) = \frac{1}{1-\eta} x_2(t)^{1-\eta}$. The planner has to find the solution to the following problem:

$$\max \int_0^\infty e^{-(\rho-n)t} \frac{1}{1-\eta} x_2(t)^{1-\eta}\, dt \tag{23}$$

$$\text{s.t. } \dot{k}_1 = \lambda A_1(t) k_1^\alpha a^{1-\alpha} - (n+\delta) k_1 \tag{21'}$$

$$\dot{k}_2 = (1-\lambda) A_1(t) k_1^\alpha a^{1-\alpha} - (n+\delta) k_2 \tag{22'}$$

$$1 \geq \lambda(t) \geq 0 \tag{24}$$

$$[k_1(0), k_2(0)] = [k_1^0, k_2^0] \tag{25}$$

where ρ is the social rate of pure time discount and η is the absolute value of the elasticity of marginal utility. Introducing two co-state variables or undiscounted non-negative prices $q_1(t)$ and $q_2(t)$ to the investment in both sectors, we arrive at the following Hamiltonian:

$$H = \frac{1}{1-\eta} x_2(t)^{1-\eta} + q_1(t)[\lambda A_1(t) k_1^\alpha a^{1-\alpha} - (n+\delta) k_1] + q_2(t)[(1-\lambda)\sigma A_1(t) k_1^\alpha a^{1-\alpha} - (n+\delta) k_2] \tag{26}$$

The relevant first-order condition (FOC), characterized by the maximum principle, may be written as:

$$H_\lambda = 0 \Rightarrow a^{1-\alpha} k_1^\alpha A_1(t)[q_1 - q_2] = 0 \tag{27}$$

The Euler equations are given by:

$$\dot{q}_1 = \left[n + \delta + \rho - \lambda A_1(t)a^{1-\alpha}k_1^{\alpha-1}\alpha\right]q_1 - (1-\lambda)A_1(t)a^{1-\alpha}k_1^{\alpha-1}\alpha q_2 \quad (28)$$

$$\dot{q}_2 = (n + \delta + \rho)q_2 - [A_2 k_2^{\beta}(1-a)^{1-\beta}]^{1-\eta}\beta k_2^{-1} \quad (29)$$

The transversality conditions are given by:

$$\lim_{t\to\infty} q_1(t)e^{-\rho t}k_1(t) = \lim_{t\to\infty} q_2(t)e^{-\rho t}k_2(t) = 0 \quad (30)$$

$$\lim_{t\to\infty} q_1(t) \geq 0, \lim_{t\to\infty} q_2(t) \geq 0 \quad (31)$$

Expression (27), or the FOC yields:

$$q_1 = q_2 \quad (32)$$

This expression shows that the optimal policy consists in equalizing the opportunity costs – or shadow price – of investment in sectors 1 and 2. If $q_1 > q_2$, for instance, then the opportunity cost of investment in sector 1 is larger than in the sector 2 and we are not in an optimal solution. Substituting Expression (32) into Expression (28) it yields:

$$\dot{q}_1 = (n + \ \delta + \ \ \rho \ A_1(t)a^{1-\alpha}k_1^{\alpha 1}\alpha)q_1 \quad (28)'$$

Then we are to prove the following:

- *Proposition 2*: In steady state, the growth rate of the capital stock in sector 1 is given by $\frac{\dot{k}_1}{k_1} = m(1- \ \alpha)^{-1}$ q.e.d.

• *Proof*: Taking the derivative with respect to time of Expression (29) evaluated in steady state, it yields the following relation between the growth rate of the state, k_2, and co-state, q_2, variables: $\frac{\dot{k}_2}{k_2} = -\frac{\dot{q}_2}{q_2}$. Considering that in steady state $\frac{\dot{k}_2}{k_2} = \frac{\dot{k}_1}{k_1}$ and $\frac{\dot{q}_1}{q_1} = \frac{\dot{q}_2}{q_2}$ we conclude that $\frac{\dot{k}_1}{k_1} = -\frac{\dot{q}_1}{q_1}$. Then by equalizing Expression (21)' divided by k_1 to the symmetric of Expression (28)' divided by q_1, we obtain after some algebraic manipulation the dynamic path of the capital stock: $k_1(t) = a\left(\frac{A_1(t)(\alpha - \lambda)}{\rho}\right)^{\frac{1}{1-\alpha}}$. As we are considering that $A_1(t) = A_1 e^{mt}$, taking log and differentiating both sides of Expression (33) allows us to conclude that the growth rate of the capital stock in sector 1 is given by: $\frac{\dot{k}_1}{k_1} = m(1-\ \alpha)^{-1}$.

But, from Expression (28)', evaluated in steady state we conclude that:

$$k_1{}^*(t) = a\left(\frac{\alpha A_1(t)}{m(1-\alpha)\ + n + \delta + \rho}\right)^{\frac{1}{1-\alpha}} \qquad (33)$$

By equalizing Expression (33) to $k_1(t) = a\left(\frac{A_1(t)(\alpha - \lambda)}{\rho}\right)^{\frac{1}{1-\alpha}}$, it is possible to determine the value of λ associated to the steady state of the system:

$$\lambda^* = \frac{\alpha[m(1-\alpha)\ + n + \delta]}{m(1-\alpha)\ + n + \delta + \rho} \qquad (34)$$

This solution is akin to what Domar (1957) considered the aim of planners, namely to choose the optimum path, depending on their own objective, and on their evaluation of existing economic and political conditions and possibilities. From (22)', (33) and (34) we obtain the steady-state value of k_2:

$$k_2{}^* = \frac{a\left[(1-\alpha)[m(1-\alpha)+n+\delta+\rho\right]}{m(1-\alpha)+n+\delta+\rho}\left(\frac{\alpha A_1(t)}{m(1-\alpha)+n+\delta+\rho}\right)^{\frac{1}{1-\alpha}} \quad (35)$$

Now it is possible to conclude from expressions (33) and (35) that the steady state dynamic path solution for the stock of capital in sectors 1 and 2 will take into account the social rate of pure time discount, namely ρ, expressing the fact that now the discount of future consumption will be considered to determine the optimal value for the capital stock in both sectors 1 and 2.

The advantage of our approach is that during the transition phase until the steady state be reached, it is possible to consider the structural changes in the model due to variations in the rate of investment allocation. In fact the concepts of convergence and steady state are closely related in this new version. In order to study this connection, let us follow Weitzman (1971) and define an auxiliary variable as:

$$x(t) \equiv \frac{k_1(t)}{k_2(t)}; \quad x^o \equiv \frac{k_1^0}{k_2^0} \quad (36)$$

The steady state the value of $x(t)$ is given by:

$$x^* = \frac{k_1^*}{k_2^*} = \frac{(1-\alpha)\left[m(1-\alpha)+n+\delta\right]+\rho}{m(1-\alpha)+n+\delta+\rho} \quad (37)$$

Assume now, for instance, that:

$$x^o < x^* \quad (38)$$

This means that the stock of capital in the capital goods sector compared to that in the consumption goods is smaller than what is necessary to reach the steady state level x^*. In this case, to achieve the steady-state path the initial phase of the optimal program requires that $\frac{\dot{k}_1}{k_1} > \frac{\dot{k}_2}{k_2}$, until the moment that x reaches x^*. This condition can be rewritten as:

$$\lambda A_1 k_1^{\alpha-1} a^{1-\alpha} - n > (1-\lambda) A_1 k_1^{\alpha-1} x a^{1-\alpha} - n \tag{39}$$

After some algebraic manipulation it is possible to show that this condition is equivalent to:

$$\lambda(t) > \frac{x(t)}{1+x(t)} \tag{40}$$

That is, the value of λ must lie in the following interval, $\lambda(t) \in \left(\frac{x(t)}{1+x(t)}, 1\right]$, in other to guarantee that the convergence path is consistent with the steady state path of the economy while $x^o < x^*$. This result suggests a policy in terms of specialization in the sector, in this case sector 1, which has less capital than what is required to achieve the steady state path. Note that this does not require a full specialization is this sector. However a value of λ near to one in this case will lead fast the value of $x(t)$ to x^*. These results are in accordance with Feldman's conclusions. After x reaches x^* the steady state value of λ is given by Expression (34). The complete characterization of the optimal program can be described as follows:

$$k_2^* = \frac{a\left[(1-\alpha)[m(1-\alpha)+n+\delta+\rho]\right]}{m(1-\alpha)+n+\delta+\rho}\left(\frac{\alpha A_1(t)}{m(1-\alpha)+n+\delta+\rho}\right)^{\frac{1}{1-\alpha}} \tag{35}$$

$$\lambda(t)\begin{cases} \in\left(\dfrac{x(t)}{1+x(t)},1\right] & \text{if } \ x(t) < x^* \\ = \dfrac{\alpha[m(1-\alpha)+n+\delta]}{m(1-\alpha)+n+\delta+\rho} & \text{if } \ x(t) = x^* \\ \in\left[0,\ \dfrac{x(t)}{1+x(t)}\right) & \text{if } \ x(t) > x^* \end{cases} \tag{41}$$

Note that if $x(t) = x^*$ then necessarily $\lambda^* = \dfrac{\alpha[m(1-\alpha)+n+\delta]}{m(1-\alpha)+n+\delta+\rho}$ to keep the balanced growth path.

4 CONCLUDING REMARKS

One of the main characteristics of F-M approach to investment allocation is that it disregards demand requirements. As pointed out by Halevi (1996, p. 163) "the limitations of the Feldman-Mahalanobis model of growth emerge in relation to the passive role of per capita consumption demand". In order to overcome these limitations, a normative criterion for this model was introduced here: the optimal rate of investment allocation was determined subject to the intertemporal maximization of consumption.

Following this line, we extended the dynamic intertemporal version of the F-M two-sector growth model to the case of a decentralized competitive equilibrium. This was accomplished by substituting Leontief production functions for Cobb-Douglas production functions in both sectors. It is shown that decentralized markets can mimic the dynamic behavior of the centrally planned economy with two sectors. In this vein we have proved the Pareto Optimality of the outcomes of the analysis of investment allocation by using Cobb-Douglas technology. This result does not hold for the original Feldman's model due to the labor market failure. Moreover, we show that the steady state solutions are stable, which is in sharp contrast with the F-M model in which the equilibrium solutions are unstable.

With this approach we intend to highlight the importance of the approach of investment allocation initiated by Feldman (1928) and carried out by a

number of authors. In this vein we show that this analysis is not confined to the case of a Leontief technology but may be extended to consider other kinds of technologies such as the Cobb-Douglas production function. Arguably, the analysis of investment allocation is shown to be robust and its relevance proved to have a broader relevance than what was originally thought by its authors.

ALOCAÇÃO DE INVESTIMENTO IDEAL EM MERCADOS DESCENTRALIZADOS

Resumo

Este artigo faz três contribuições ao modelo de alocação de investimentos de Feldman-Mahalanobis. Em primeiro lugar, ele supera a limitação do modelo original, que assume um papel passivo de demanda de consumo, por meio da introdução de maximização intertemporal do consumo. Em segundo lugar, mostra que os mercados descentralizados podem imitar o comportamento dinâmico da economia centralmente planejada com dois setores, um de bens de consumo e outro de bens de investimento. Isso é conseguido por meio da utilização de funções de produção Cobb-Douglas em ambos os setores. Em terceiro lugar, em contraste com o modelo original em que as soluções são instáveis, este artigo prova a estabilidade das soluções em estado estacionário.

Palavras-chave: Alocação de investimentos; Modelos de dois setores; Otimização dinâmica.

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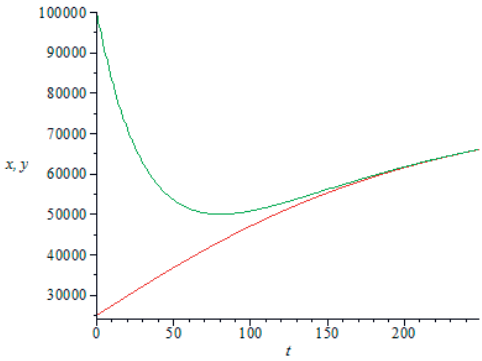
Appendix

Here we depict the behavior of the dynamical system formed by expressions (9)' and (11)' when $\lambda = \frac{1}{2}$, $n = 0.03$ $\lambda = \frac{1}{2}$, $n = 0.03$ and $\alpha = 0.75$.

Graph1

The dynamic paths for $[k_1(t), k_2(t)]$ for the solution of the system (9)' and (11)' with initial condition $[k_1(0), k_2(0)] = [25000{,}10000]$

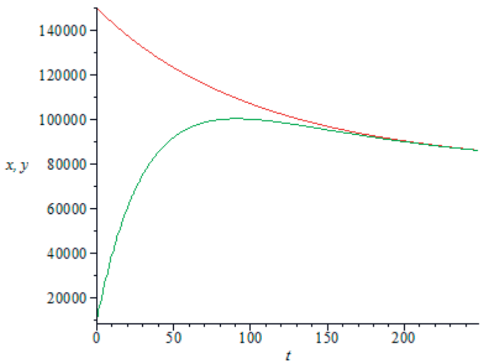


Note that both dynamic paths are increasing.

Source: Elaborated by the authors.

Graph 2

The dynamic paths for $[k_1(t), k_2(t)]$ for the solution of the system (9)' and (11)' with initial condition $[k_1(0), k_2(0)] = [25000{,}100000]$



Note that while $k_1(t)$ increases, $k_2(t)$ decreases.

Source: Elaborated by the authors.