# A POST KEYNESIAN MODEL OF GROWTH AND WEALTH DISTRIBUTION: GOVERNMENT'S OPTIMAL TAX CHOICE USING AN INTERTEMPORAL INFINITELY REPRESENTATIVE AGENT

*UM MODELO PÓS-KEYNESIANO DE CRESCIMENTO E DISTRIBUIÇÃO DE RENDA: A QUESTÃO DA TAXAÇÃO ÓTIMA DO GOVERNO COM AGENTE MAXIMIZADOR INTERTEMPORAL INFINITO*

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## Resumo

O artigo avalia como o modelo pós-keynesiano de crescimento e distribuição de renda da linha Kaldor-Pasinetti pode ser estendida usando-se o agente maximizador intertemporal infinito do tipo Ramsey (1928). Verifica-se que essa modelagem possibilita a inclusão da variável lazer como qualificadora de classes sociais e, ademais, permite que o governo busque uma taxação ótima sobre os lucros e os salários, tendo em vista que os agentes definidos na economia respondem às políticas fiscais. Os resultados mostram que o teorema dual continua inválido, a propensão a poupar dos capitalistas é endógena e, dada a especificidade para essa economia, a taxação sobre os ganhos de capital é zero em equilíbrio de longo prazo.


**Palavras-chave**: Crescimento econômico; Distribuição de renda; Modelos intertemporais.


## Abstract

The article evaluates how a post Keynesian model of growth and wealth distribution in the line of Kaldor-Pasinetti can be extended using an infinitely lived representative agent of the type of Ramsey (1928). It is verified that the model permits the inclusion of leisure as a variable class distinctive as well as permits the government to choose optimal taxes on profits and on wages once the economic agents defined for this economy respond to fiscal policies. The results show that the Dual Theorem does not hold, the marginal propensity to save of the capitalists is endogenous and, given the specificity for this economy, the tax on the gain from capital is zero in the long run equilibrium.


**Keywords**: Economic growth; Wealth distribution; Intertemporal models.

# 1 INTRODUCTION

The Introduction of the public sector, i.e., of government taxation and expenditure, is a line of research that has been taken up by several authors within the framework of the standard two-class post Keynesian model of economic growth, income distribution and capital accumulation. In this paper we intend to provide a complete characterization of the fiscal policy by determining the optimal taxation on capital income and wages. Turnovsky (1989, p. 342) points out that "the theory of optimal taxation is one of the central issues in public finance".

Here we approach this issue by taking the alternative route of using an intertemporal representative agent model with Pasinettian features. We postulate the existence of one representative capitalist and worker. From the interaction between the public sector optimal policy and the optimal decision from workers and capitalists, it is possible to fully characterize the optimal fiscal policy, through the determination of the optimal taxes on capital income and wages.

We begin by solving a capitalist and a worker's maximization problem using an infinitely lived representative agent. The variables introduced in this step enable a better distinction of social classes for this economy in a vein suggested by Baranzini (1991, p. xv). Furthermore, the public sector optimal policy will take such a difference into consideration through the maximization of capitalist and worker's indirect utility function.

The dynamic path of optimal taxes on capital income in general equilibrium models was studied by many influential authors. Chamley (1985, 1986) and Judd (1985) showed that the optimal tax rate on capital income is zero in the long run. Other papers such as those of Jones, Manuelli and Rossi (1997) generalized the approach including human capital and found that a more useful result would be a non-zero tax on capital income.

According to Chamley (1986, p. 607): "[…] when individuals have infinite lives and a utility function of a general form, the optimal tax rate in capital income does tend to zero in the long run". In other words, given the characteristics of the economy set up in this paper the result obtained is likewise: zero optimal tax rate on the gains from capital in the long run and a positive value on the gains from labor.

The model shows other important results. The extension of the Faria and Araujo's paper (2004) invalidates the controversial dual theorem. Moreover, the rate of profit is not determined by the Cambridge equation. As showed, the Cambridge equation now determines the equilibrium value of the capitalist's marginal propensity to save. Once more, neither the marginal product of capital nor any parameter of the production function affects the capitalist's marginal propensity to save. Another result showed the strength of the rate of time preference in determining the patter of accumulation.

Including this introduction, the paper is organized as follows. In section 2, we define the main features for this economy. In section 3, the public sector is introduced in the economy where it aims the optimal taxes on profit and wages. In section 4, we offer some concluding comments.

# 2 THE MODEL

Our economy consists of two institutions: firms and households. The households are further separated into two classes: the capitalists, represented by $Z$, whose source of income is uniquely gains from capital and, workers, represented by $N$, whose sources of incomes come from capital as well as from labor. The worker-population ratio is given by a fixed value $b$. Then, given that the population is represented by $L$, we have:

$$L = Z + N \tag{1}$$

$$N = bL \tag{2}$$

$$0 < b < 1 \tag{3}$$

The population grows at an exogenous rate $n$:

$$\frac{\dot{L}}{L} = n \Rightarrow L = \exp[nt] \tag{4}$$

Given the equations above, from (1) to (4), both classes will be growing exactly at the same rate $n$.

Total real output is produced by labor $N$ and the total quantity of capital $K$, which is by its turn separated into a part belonging to the capitalists, $K_c$, and a second part belonging to the workers, $K_w$. Therefore, our well behaved and constant return to scale production function is:

$$Y = F(K, N) \tag{5}$$

$$K = K_w + K_c \tag{6}$$

Furthermore, the share of capital owned by workers is given by $a$. That is:

$$K_w = aK \tag{7}$$

Using lower case letters to rewrite the variables in per capita terms, we have:

$$y = \frac{Y}{L} = F(k,b) = F(k_c + k_w, b) \tag{8}$$

The market structure for firms is said to be competitive. It, therefore, means that they take the rental price of capital as well as the rental price of labor as given to produce output. The first one is represented by the interest rate or rate of profit, $r$, and the second one is denoted by $w$. Profit maximization implies that:

$$r = F_k(k,b) \tag{9}$$

$$bw = F(k,b) - rk \tag{10}$$

Where (9) is the marginal productivity of capital. Thus, the total income of the economy in per capita terms is given by:

$$y = bw + r(k_w + k_c) \tag{11}$$

The capitalist's problem is to maximize a class-representative intertemporal utility function constrained by a class-representative dynamic budget constraint:

$$Max \int_0^\infty V(c_c)\exp[-(\theta - n)t]dt \tag{12}$$

$$\text{Subject to } \dot{k}_c = (1-t_p)rk_c - c_c - nk_c \tag{13}$$

Where the parameter $\theta$ is the rate of time preference and assumed to be strictly positive. $V(\cdot)$ is a concave, instantaneous utility function of the capitalist and increasing function of the total consumption, $c_c$, defined as:

$$c_c = \frac{C_c}{L} = \frac{C_c}{Z}\cdot\frac{Z}{L} = (1-b)\frac{C_c}{Z} \tag{14}$$

Notice here that we have already considered the intervention of the public sector in the form of a tax rate on profit, $t_p$. The relevant first order conditions imply that in steady state the following results hold:

$$(a)\quad r = \frac{\theta}{(1-t_p)} \tag{15}$$

$$(b)\quad c_c = (1-t_p)rk_c - nk_c \tag{16}$$

$$(c)\quad s_c = \frac{n}{(1-t_p)\cdot r} \tag{17}$$

Where, equation (17) is the capitalist's marginal propensity to save. As can be noted, the higher the tax rate on profit the higher will be the capitalist's saving. Once the government chooses such tax, the Cambridge Equation determines the rate of saving for this economy and not the rate of profit, given by equation (15), which will be determined now by the rate of time preference, $\theta$, and $t_p$, the profit tax rate.

The worker's problem is also to maximize a class-representative intertemporal utility function. But, in the present case, we include a class-distinctive variable, leisure $l$. The constraint is again a class-representative dynamic budget.

$$Max \int_0^{\infty} U(c_w, l)\exp[-(\theta - n)t]dt \tag{18}$$

$$\dot{k}_w = (1-t_p)rk_w + (1-t_w)wb(1-l) - c_w - nk_w \tag{19}$$

Once more, $\theta$, is the rate of time preference and strictly positive. $U(\cdot)$ is a concave, instantaneous utility function and an increasing function of the total consumption given by $c_w$ and defined as:

$$c_w = \frac{C_w}{L} = \frac{C_w}{N} \cdot \frac{N}{L} = \frac{C_w}{N} \cdot b \tag{20}$$

To solve this intertemporal maximization problem, we define a Cobb-Douglas form utility function:

$$U(c_w, l) = \left(\frac{c_w}{N} b\right)^{\alpha} \cdot l^{1-\alpha} \tag{21}$$

The relevant first order conditions imply that in steady state:

$$(a) \quad r = \frac{\theta}{(1-t_p)} \tag{22}$$

$$(b') \quad c_w = [r(1-t_p) - n]k_w + (1-t_w)wb(1-l) \tag{23}$$

$$(c') \quad s_w = \frac{nk_w}{r(1-t_p)k_w + w(1-t_w)b(1-l)} \tag{24}$$

$$(d) \quad l = \frac{(1-\alpha)}{(1-t_w)wb} \tag{25}$$

With such equations, we reached a more robust result for this economy:

($i$) The rate of profit, $r$, is not determined by the Cambridge Equation any longer. It will now be determined by the rate of time preference, $\theta$;

($ii$) The Cambridge Equation was the key element to endogenize the capitalist's marginal propensity to save, $s_c$;

($iii$) The Dual Theorem or "Anti-Pasinetti" remains invalid:

$$s_c = \frac{n}{(1-t_p)\cdot r} > s_w = \frac{n}{\dfrac{b(1-l)(1-t_w)w}{k_w} + (1-t_p)\cdot r} \qquad (26)$$

($iv$) The equation for the supply of labor works accordingly to the economic theory:

$$\frac{\partial l}{\partial w} = -\frac{(1-\alpha)}{(1-t_w)w^2 b} < 0 \qquad (27)$$

$$\frac{\partial l}{\partial t_w} = \frac{(1-\alpha)}{(1-t_w)^2 wb} > 0 \qquad (28)$$

In other words, in steady state equilibrium, workers diminish the time allocated to leisure as long as the wage increases. On the other hand, the tax on wage, $t_w$, makes workers more willing to spend time in leisure activities rather than on work.

# 3 THE GOVERNMENT'S OPTIMAL TAX CHOICE

In this section we evaluate the validity of the results found above considering a government sector with a specific intertemporal utility function. In this path we give the public sector a more active role once it chooses how to tax different social-classes of the economy taking into account a long run growth and wealth distribution.

In fact, as stated previously, the introduction of the government sector in the Kaldor-Pasinetti models complies with John M. Keynes' main idea for a non-neutral public sector (BARANZINI, 1991, p. 61) and this can be done using alternative modeling such as a balanced budget or not, whether the government accumulates capital or not, the different mechanism for obtaining revenues and financing expenditures and others (FARIA; ARAUJO, 2004, p. 261).

We begin by solving the firms' profit maximization problem. The idea is to attain some fundamentals variables needed to solve the equilibrium for this economy. In other words, prior to finding an optimal $t_p$ and $t_w$, we must first find $r$, $l$ (already obtained), $w$ and $k$.

The firms' profit equation is:

$$\Pi = P \cdot Y[K, N(1-l)] - wN(1-l) - rK \tag{29}$$

Where: $P$ is the price;

$Y[\cdot]$ is the production function;

$K$ is the total quantity of capital;

$N$ is the labor force;

$l$ is the part of the labor force that chose to spend time on leisure activity rather than on job.

Additionally, we assume that:

$$\frac{Y}{L} = \frac{F}{L}\left(\frac{K}{L}, \frac{N(1-l)}{L}\right) \tag{30}$$

Therefore, specifying the production function and assuming $P = 1$:

$$\Pi = k^{\gamma} N^{1-\gamma}(1-l)^{1-\gamma} - wN(1-l) - rK \tag{31}$$

Finally, the first order conditions with respect to and $k$ and $N$ give the following equations (see the appendix):

$$w^* = \left( \frac{\theta}{(1-t_p)\gamma} \right)^{\frac{\gamma}{\gamma-1}} \cdot (1-\gamma) \qquad (32)$$

$$k^* = \frac{\gamma}{(1-\gamma)} \cdot \frac{(1-t_p)}{\theta} \cdot b \left\{ \frac{(\alpha-1)}{(1-t_w)} + b \left[ \frac{\theta}{(1-t_p)\gamma} \right]^{\frac{\gamma}{\gamma-1}} (1-\gamma) \right\} \qquad (33)$$

The representative government maximizes the following intertemporal utility function:

$$Max \int_0^\infty [V(c_c)+U(c_w,l)]\exp[-(\theta-n)t]dt \qquad (34)$$

$$\text{Subject to: } \dot{k} = f(K)-c-g-nk \qquad (35)$$

Where:

$$f(k) = k^\gamma \cdot (1-l)^{1-\gamma} \cdot b^{1-\gamma}$$

$g$ is the government expenditure and considered exogenous;

$$c_c = [(1-t_p)r-n]k_c$$

$$c_w = [(1-t_p)r-n]k_w + (1-t_w)w(1-l)$$

Linearizing the consumption functions:

$$V(c_c) = \ln\{[(1-t_p)r-n]k_c\} \qquad (36)$$

$$U(c_w) = \ln\{[(1-t_p)r-n]k_w + (1-t_w)wb(1-l) \qquad (37)$$

The Hamiltonian for the government maximization is:

$$\begin{aligned} H &= \ln\{[(1-t_p)r-n]k_c\} + \ln\{[(1-t_p)r-n]k_w + (1-t_w)wb(1-l) \\ &+\mu_t\{k^{\gamma}(1-l)^{1-\gamma}b^{1-\gamma} - [(1-t_p)r-n]k_c - [(1-t_p)r-n]k_w \\ &-(1-t_w)wb(1-l) - g - nk\} \end{aligned} \tag{38}$$

Given that $t_p$ and $t_w$ are the control variables, the first orders conditions are:

$$\begin{aligned} &(i) - H_{t_p} = 0 \\ &(ii) - H_{t_w} = 0 \\ &(iii)\, \dot{\mu}_t - \mu_t(\theta - n) = -H_k \end{aligned} \tag{39}$$

We, therefore, obtain (see the appendix):

$$r = \theta \tag{40}$$

Finally, the equation (40) shows that in steady state the government chooses not to tax the profit. That is, comparing the result obtained previously in equation (22), the new equilibrium results now in $t_p = 0$. The solution for the whole economy requires us to find $t_w$ Thus, applying the condition found, $t_p = 0$, we find (see the appendix):

$$t_w = 1 - \frac{\frac{\gamma}{(1-\gamma)} \cdot \frac{1}{\theta}\left[b(\alpha-1)+(1-\alpha)\right]}{\left(\frac{\theta}{\gamma}\right)^{\frac{1}{\gamma-1}} b(1-b)} \tag{41}$$

It is important to note that the tax on wage varies in an interval between 0 and 1 even considering a zero tax on profit. Moreover, it is easily observed that $t_w$ depends only on the parameters defined for this economy.

# 4 CONCLUDING REMARKS

One of the main lines of research of post Keynesian economics is to provide some micro-foundations for the two-class model of growth and income distribution (BARANZINI, 1991, p. 107). Specifically in this model, we believe the instrument of an infinitely lived representative agent is a very handy way of dealing with variables such as those in the maximization process carried out by the government.

In other words, the introduction of more variables class-distinctive gives the public sector an easier and, at the same time, a more realistic role. As it was demonstrated, agents' behaviour can vary accordingly to the fiscal policies. Workers can supply more or less jobs depending upon how high is the tax rate on wages and, therefore, turns the government's role much more interesting.

The achievement of a zero-tax rate on the gains from capital resulted directly from the definition for this economy, a result already antecipated by Chamley (1986), and provided an alternative way of including the post Keynesian models toward theories of optimal taxation. Besides, even in a "hybrid" model, where such an instrument like the Ramsey (1928) representative agent was applied, it has satisfactorily been shown that one central issue of that school remained the same, whilst others could be extended successfully.

The Dual Theorem does not hold, the rate of time preference determines the rate of profit and the Cambridge Equation determines the marginal propensity of the capitalists. The first one implies the impossibility of a higher marginal propensity to save of workers compared to that of the capitalists'. Thus, the pattern of accumulation for this economy rules out the case called the "capitalist's euthanasia".

The second result shows that for both social classes the higher the preference for present consumption the higher the rate of profit. Interestingly, such a result is totally independent of the tax on the rate of profit and, therefore, tax on wage does not affect the economy's capital accumulation. The third result endogenized capitalists' marginal propensity to save and showed that it neither depended on the marginal product of capital nor on any other parameter of the production function.

# 5 APPENDIX

a) *Solution for the problem of profit maximization*

$$\frac{\partial \Pi}{\partial k} = \gamma \cdot k^{\gamma-1} \cdot N^{1-\gamma} \cdot (1-l)^{1-\gamma} - r = 0$$
$$\frac{\partial \Pi}{\partial k} = \gamma \cdot k^{\gamma-1} \cdot N^{1-\gamma} \cdot (1-l)^{1-\gamma} = r \qquad \text{(a.1)}$$

And

$$\frac{\partial \Pi}{\partial N} = k^{\gamma} \cdot (1-\gamma) N^{-\gamma} \cdot (1-l)^{1-\gamma} - w(1-l) = 0$$
$$\frac{\partial \Pi}{\partial N} = k^{\gamma} \cdot (1-\gamma) N^{-\gamma} \cdot (1-l)^{1-\gamma} = w(1-l) \qquad \text{(a.2)}$$

Rearranging (a.1):

$$\gamma \cdot k^{-1} \cdot N \cdot k^{\gamma} \cdot N^{-\gamma} \cdot (1-l)^{1-\gamma} = r \qquad \text{(a.3)}$$

Rearranging (a.2)

$$k^{\gamma} \cdot N^{-\gamma} \cdot (1-l)^{-\gamma} = \frac{w}{(1-\gamma)} \qquad \text{(a.4)}$$

Substituting (a.4) in (a.3), we have:

$$\frac{\gamma}{(1-\gamma)} \cdot k^{-1} \cdot N \cdot w(1-l) = r$$

Once variables are in per capita terms, then: $\frac{N}{L} = b$ . Therefore:

$$\frac{\gamma}{(1-\gamma)} \cdot k^{-1} \cdot b \cdot w(1-l) = r \qquad \text{(a.5)}$$

Applying the same rule to (a.3) and rearranging it:

$$k = \left[ \frac{r \cdot b^{\gamma-1} \cdot (1-l)^{\gamma-1}}{\gamma} \right]^{\frac{1}{\gamma-1}}$$

Considering that: $\frac{1}{(\gamma-1)} = a$, we have:

$$k^{-1} = \left[ \frac{r^{a} \cdot b \cdot (1-l)}{\gamma^{a}} \right]^{-1} \qquad \text{(a.6)}$$

Substituting (a.6) in (a.5):

$$\frac{\gamma}{(1-\gamma)} \cdot bw(1-l) \cdot \left[ \frac{r^{a} \cdot b \cdot (1-l)}{\gamma^{a}} \right]^{-1} = r$$

$$\frac{\gamma}{(1-\gamma)} \cdot bw(1-l) \cdot \frac{\gamma^{a}}{r^{a} \cdot b(1-l)} = r$$

Finally, simplifying:

$$w = \frac{r^{1+a} \cdot (1-\gamma)}{\gamma^{1+a}}$$

Or

$$w^{*} = \left(\frac{r}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \cdot (1-\gamma) \qquad \text{(a.7)}$$

In order to obtain $k^*$, we substitute (a.7) in (a.5). However, we first rewrite (a.5) considering the equation (25). Thus:

$$\frac{\gamma}{(1-\gamma)} \cdot k^{-1} \cdot b \cdot w\left(1-\frac{(1-\alpha)}{(1-t_{w})wb}\right)=r$$

$$k=\frac{\gamma}{r(1-\gamma)} \cdot b\left[1-\frac{(1-\alpha)}{(1-t_{w})wb}\right] \cdot w$$

Simplifying:

$$k=\frac{\gamma}{r(1-\gamma)} \cdot b\left[wb+\frac{(\alpha-1)}{(1-t_{w})}\right]$$

Finally:

$$k^{*}=\frac{\gamma}{r(1-\gamma)} \cdot b\left[\frac{(\alpha-1)}{(1-t_{w})}+b \cdot \left(\frac{r}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \cdot (1-\gamma)\right]$$

which is exactly the equation (32).

b) *Solving the government's Hamiltonian:*

Solving the condition (ii):

$$-H_{t_w} = -\left\{ \frac{-wb(1-l)}{[(1-t_p)r-n]k_w + (1-t_w)wb(1-l)} + wb(1-l)\cdot\mu_t \right\} = 0$$

We attain:

$$\mu_t = \frac{1}{[(1-t_p)r-n]k_w + (1-t_w)wb(1-l)}$$

Solving the Euler's equation, (iii), we have:

$$\dot{\mu}_t - (\theta-n)\mu_t = -\mu_t\cdot\gamma\cdot k^{\gamma-1}\cdot(1-l)^{1-\gamma}\cdot b^{1-\gamma} + n\mu_t$$
$$\dot{\mu}_t = -\mu_t\cdot\gamma\cdot k^{\gamma-1}\cdot(1-l)^{1-\gamma}\cdot b^{1-\gamma} + n\mu_t + (\theta-n)\mu_t$$
$$\dot{\mu}_t = -\mu_t[\gamma\cdot k^{\gamma-1}\cdot(1-l)^{1-\gamma}\cdot b^{1-\gamma} + \theta]$$

Or:

$$\frac{\dot{\mu}_t}{\mu_t} = -\gamma\cdot k^{\gamma-1}\cdot(1-l)^{1-\gamma}\cdot b^{1-\gamma} + \theta$$

As in steady state: $\frac{\dot{\mu}_t}{\mu_t} = 0$, therefore:

$$\gamma\cdot k^{\gamma-1}\cdot(1-l)^{1-\gamma}\cdot b^{1-\gamma} = \theta \qquad \text{(b.1)}$$

Notice that the equation (a.2) was:

$$\frac{\partial \Pi}{\partial k} = \gamma \cdot k^{\gamma-1} \cdot N^{1-\gamma} \cdot (1-l)^{1-\gamma} = r$$

Hence, r = $\theta$

Is an automatic result for our model.

c) *Finding $t_w$: substituting the steady state level of $k^*$ and $l^*$ in equation (b.1)*

Regarding from (b.1):

$$\gamma \cdot k^{\gamma-1} \cdot (1-l)^{1-\gamma} \cdot b^{1-\gamma} = \theta$$

With $t_p$ = 0, $k^*$ and $l^*$ become:

$$\gamma \cdot \left[ \frac{\gamma}{(1-\gamma)} \cdot \frac{b}{\theta} \cdot \frac{(\alpha-1)}{(1-t_w)} + \frac{\gamma}{(1-\gamma)} \cdot \frac{b^2}{\theta} \cdot \left[ \frac{\theta}{\gamma} \right]^{\frac{\gamma}{\gamma-1}} \cdot (1-\gamma) \right]^{\gamma-1}$$
$$\cdot \left[ 1 - \frac{(1-\alpha)}{(1-t_w)b} \cdot \left( \frac{\gamma}{\theta} \right)^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{(1-\gamma)} \right]^{1-\gamma} \cdot b^{1-\gamma} = \theta$$

Rewriting:

$$\frac{\gamma \cdot \left[ \frac{\gamma}{(1-\gamma)} \cdot \frac{b}{\theta} \cdot \frac{(\alpha-1)}{(1-t_w)} + \frac{\gamma}{(1-\gamma)} \cdot \frac{b^2}{\theta} \cdot \left[ \frac{\theta}{\gamma} \right]^{\frac{\gamma}{\gamma-1}} \cdot (1-\gamma) \right]^{\gamma-1} \cdot b^{1-\gamma}}{\left[ 1 - \frac{(1-\alpha)}{(1-t_w)b} \cdot \left( \frac{\gamma}{\theta} \right)^{\frac{\gamma}{\gamma-1}} \cdot \frac{1}{(1-\gamma)} \right]^{\gamma-1}} = \theta$$

Elevating both side by $\frac{1}{(\gamma-1)}$, we have:

$$\frac{\gamma^{\frac{1}{\gamma-1}}\cdot\left[\frac{\gamma}{(1-\gamma)}\cdot\frac{b}{\theta}\cdot\frac{(\alpha-1)}{(1-t_w)}+\frac{\gamma}{(1-\gamma)}\cdot\frac{b^2}{\theta}\cdot\left[\frac{\theta}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}\cdot(1-\gamma)\right]\cdot b^{\frac{1-\gamma}{\gamma-1}}}{\left[1-\frac{(1-\alpha)}{(1-t_w)b}\cdot\left(\frac{\gamma}{\theta}\right)^{\frac{\gamma}{\gamma-1}}\cdot\frac{1}{(1-\gamma)}\right]}=$$

Rewriting:

$$\frac{\left[\frac{\gamma}{(1-\gamma)}\cdot\frac{b}{\theta}\cdot\frac{(\alpha-1)}{(1-t_w)}+\frac{\gamma}{(1-\gamma)}\cdot\frac{b^2}{\theta}\cdot\left[\frac{\theta}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}\cdot(1-\gamma)\right]}{\left[1-\frac{(1-\alpha)}{(1-t_w)b}\cdot\left(\frac{\gamma}{\theta}\right)^{\frac{\gamma}{\gamma-1}}\cdot\frac{1}{(1-\gamma)}\right]}=\frac{\gamma^{\frac{1}{\gamma-1}}}{b^{\frac{1-\gamma}{\gamma-1}}\cdot\gamma^{\frac{1}{\gamma-1}}}.$$

$$\frac{\gamma}{(1-\gamma)}\cdot\frac{b}{\theta}\cdot\frac{(\alpha-1)}{(1-t_w)}=\frac{\theta^{\frac{1}{\gamma-1}}}{b^{\frac{1-\gamma}{\gamma-1}}\cdot\gamma^{\frac{1}{\gamma-1}}}.\left[1-\frac{(1-\alpha)}{(1-t_w)b}\cdot\left(\frac{\gamma}{\theta}\right)^{\frac{\gamma}{\gamma-1}}\cdot\frac{1}{(1-\gamma)}\right]$$
$$-\frac{\gamma}{(1-\gamma)}\cdot\frac{b^2}{\theta}\cdot\left[\frac{\theta}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}\cdot(1-\gamma)$$

$$\frac{\gamma}{(1-\gamma)}\cdot\frac{b}{\theta}\cdot\frac{(\alpha-1)}{(1-t_w)}=\frac{\theta^{\frac{1}{\gamma-1}}}{b^{\frac{1-\gamma}{\gamma-1}}\cdot\gamma^{\frac{1}{\gamma-1}}}-\frac{\theta^{\frac{1}{\gamma-1}}}{b^{\frac{1-\gamma}{\gamma-1}}\cdot\gamma^{\frac{1}{\gamma-1}}}\cdot\frac{(1-\alpha)}{(1-t_w)b}\cdot\left(\frac{\gamma}{\theta}\right)^{\frac{\gamma}{\gamma-1}}\cdot\frac{1}{(1-\gamma)}$$
$$-\frac{\gamma}{(1-\gamma)}\cdot\frac{b^2}{\theta}\cdot\left[\frac{\theta}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}\cdot(1-\gamma)$$

$$\frac{\gamma}{(1-\gamma)}\cdot\frac{b}{\theta}\cdot\frac{(\alpha-1)}{(1-t_w)}+\frac{\theta^{\frac{1}{\gamma-1}}}{b^{\frac{1-\gamma}{\gamma-1}}\cdot\gamma^{\frac{1}{\gamma-1}}}\cdot\frac{(1-\alpha)}{(1-t_w)b}\cdot\left(\frac{\gamma}{\theta}\right)^{\frac{\gamma}{\gamma-1}}\cdot\frac{1}{(1-\gamma)}=\frac{\theta^{\frac{1}{\gamma-1}}}{b^{\frac{1-\gamma}{\gamma-1}}\cdot\gamma^{\frac{1}{\gamma-1}}}$$
$$-\frac{\gamma}{(1-\gamma)}\cdot\frac{b^2}{\theta}\cdot\left[\frac{\theta}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}\cdot(1-\gamma)$$

Multiplying both sides by $(1 - t_w)$, we have:

$$\frac{\gamma}{(1-\gamma)}\cdot\frac{b}{\theta}\cdot(\alpha-1)+\frac{\theta^{\frac{1}{\gamma-1}}}{b^{\frac{1-\gamma}{\gamma-1}}\cdot\gamma^{\frac{1}{\gamma-1}}}\cdot\frac{(1-\alpha)}{b}\cdot\left(\frac{\gamma}{\theta}\right)^{\frac{\gamma}{\gamma-1}}\cdot\frac{1}{(1-\gamma)}=$$
$$\left[\frac{\theta^{\frac{1}{\gamma-1}}}{b^{\frac{1-\gamma}{\gamma-1}}\cdot\gamma^{\frac{1}{\gamma-1}}}-\frac{\gamma}{(1-\gamma)}\cdot\frac{b^{2}}{\theta}\cdot\left[\frac{\theta}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}\cdot(1-\gamma)\right](1-t_w)$$

Once we know that $b^{\frac{1-\gamma}{\gamma-1}}=b^{-1}$, isolation $(1 - t_w)$:

$$1-t_w=\frac{\frac{\gamma}{(1-\gamma)}\cdot\frac{b}{\theta}\cdot(\alpha-1)+\frac{\theta^{\frac{1}{\gamma-1}}}{\gamma^{\frac{1}{\gamma-1}}}\cdot\frac{(1-\alpha)}{(1-\gamma)}\cdot\left(\frac{\gamma}{\theta}\right)^{\frac{\gamma}{\gamma-1}}}{\frac{\theta^{\frac{1}{\gamma-1}}}{\gamma^{\frac{1}{\gamma-1}}\cdot b^{-1}}-\frac{\gamma}{(1-\gamma)}\cdot\frac{b^{2}}{\theta}\left[\frac{\theta}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}\cdot(1-\gamma)}$$

$$t_w=1-\frac{\frac{\gamma}{(1-\gamma)}\cdot\frac{b}{\theta}\cdot(\alpha-1)+\frac{\theta^{\frac{1}{\gamma-1}}}{\gamma^{\frac{1}{\gamma-1}}}\cdot\frac{(1-\alpha)}{(1-\gamma)}\cdot(\theta)^{\frac{\gamma}{\gamma-1}}}{\frac{\theta^{\frac{1}{\gamma-1}}}{\gamma^{\frac{1}{\gamma-1}}\cdot b^{-1}}-\frac{\gamma}{(1-\gamma)}\cdot\frac{b^{2}}{\theta}\left[\frac{\theta}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}\cdot(1-\gamma)}$$

Rearranging:

$$t_w = 1 - \frac{\frac{\gamma}{(1-\gamma)} \cdot \frac{b}{\theta} \cdot (\alpha - 1) + \frac{\theta}{\gamma} \cdot \frac{(1-\alpha)}{(1-\gamma)}}{\frac{\theta^{\frac{1}{\gamma-1}}}{\gamma^{\frac{1}{\gamma-1}}} \cdot b - b^2 \cdot \frac{\theta^{\frac{1}{\gamma-1}}}{\gamma^{\frac{1}{\gamma-1}}}}$$

$$t_w = 1 - \frac{\frac{\gamma}{(1-\gamma)} \cdot \frac{1}{\theta}\left[b(\alpha - 1) + (1-\alpha)\right]}{\frac{\theta^{\frac{1}{\gamma-1}}}{\gamma^{\frac{1}{\gamma-1}}} \cdot b(1-b)}$$

And, finally:

$$t_w = 1 - \frac{\frac{\gamma}{(1-\gamma)} \cdot \frac{1}{\theta}\left[b(\alpha - 1) + (1-\alpha)\right]}{\left(\frac{\theta}{\gamma}\right)^{\frac{1}{\gamma-1}} b(1-b)} \qquad \text{(c.1)}$$

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