# **Competitiveness of the Brazilian pulp industry:** a real options analysis of a forestry investment



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## ABSTRACT

Brazil faces challenges that have to be overcome in order to turn it into a productive and competitive player in the world. These challenges have come to be known as "Brazil Cost". But, in spite of that, some sectors of the Brazilian economy are so productive that not even such high premium cost can hinder their world competitiveness. The pulp industry is one of them. Due to the country's comparative advantage in the production of cellulose and the importance of the sector to the Brazilian economy, this paper analyzes a forestry investment project. We also chose to use a real options approach to evaluate the forestry investment and the optimal time to harvest, since this approach can capture the value of flexibility of the investment decision. The option that results from this modeling has no closed analytical solution and so we had to resort to a fully implicit finite difference numerical solution that leads to systems of linear equations which were solved with an iterative algorithm called Projected Successive Over Relaxation (Psor), implemented computationally with software developed for this specific purpose. The main objective of this paper which was to examine a forest investment in light of the theory of real options was reached and the optimal harvesting time was identified. In a context where the aspects of sustainability are growing in importance, identifying the optimum harvesting time enables the productivity of forests to be maximized, which in turn should decrease the environmental impact of the industry.

KEYWORDS
Forestry investment valuation. Brazilian pulp industry. Real Options. Fully implicit finite difference method. Psor algorithm.



## INTRODUCTION

The difficulties to do business in Brazil have come to be known as "Brazil Cost". The country faces several challenges that are yet to be overcome in order to turn it into a very productive and competitive player in the world. These challenges include very high interest rates, heavy regulation, a complicated and expensive tax structure, a lag of infrastructure (for example, bad highways, insufficient rail system, outdated ports, insufficient airports), an inflexible labor law, and a wide social gap. A recent study estimated in 36.27% the over cost to do business in Brazil as compared to Germany (ASSOCIAÇÃO BRASILEIRA DA INDÚSTRIA DE MÁQUINAS E EQUIPAMENTOS, 2010). But, in spite of that, some sectors of the Brazilian economy are so productive that not even such high premium cost can hinder their world competitiveness. The pulp industry is one of them.

Brazil has proved to be a strong competitor in the production of hardwood pulp. The country's large agricultural areas allow the cultivation of forests with little or no interference with food production. Furthermore the climatic conditions combined with the eucalyptus adaptation to the Brazilian soil allow crops in a very short period of time when compared to crops in other countries, making the hardwood pulp produced in the country very competitive. In 2008 Brazil produced 12.696.546 tons of pulp and rose from sixth to fourth place among world producers of pulp. In 2009 Brazil produced 13.314.873 tons of pulp, 85.4% of that being hardwood pulp. In 2010 the production is expected to have reached 14 million tons of pulp. As productivity is regarded, to produce 1.0 million tons of pulp per year it is necessary 100,000 ha of forest area in Brazil, while in the Iberian Peninsula, the same production requires 300,000 ha and in Scandinavia 720,000 ha are necessary. In 1980 the average productivity in Brazil was 24 $m^3$/ha/year and it rose steadily to reach 44$m^3$/ha/year in 2009, already a world record, but with potential to increase even more. Uruguay and Chile, which also plant eucalyptus and come in second place as productivity is concerned, produce an average of only 25 $m^3$/ha/year (ASSOCIAÇÃO BRASILEIRA DE CELULOSE E PAPEL, 2010).

Due to the country's comparative advantage in the production of cellulose and the importance of the sector to the Brazilian economy, we decided in this paper to analyze a forestry investment project – although the same type of modeling could be used to examine many different types of investment projects.

Most investment decisions share to a greater or lesser degree three important features: 1. the investment is partially or completely irreversible, 2. there is uncertainty about the future cash flows of the investment, 3. there is an interval of time for the investment decision to be made. These three characteristics have to be taken into account in determining the optimal investment decision. The traditional theory of investment analysis – that uses the net present value method – does not recognize the importance of the interaction between irreversibility, uncertainty, and optimal decision point of investment in time. Hence the decision to use a real options approach in this paper. The real option approach does not replace the net present value method, but rather complements it.

Since options encountered in practice rarely have closed analytical solutions, they often require the use of numerical methods to solve the differential equations of the option pricing models. Most papers that used real options for evaluating investments published in Brazil have relied on binomial and trinomial trees to evaluate these options. We can mention for example, among many others, Sato (2004) who studied the abandonment option of a coffee plantation using binomial trees, Nogueira (2005) who examined the same abandonment option using trinomial trees, and Tavares (2003) who studied the buyback guarantee offered by a network of franchises using binomial trees. In this article, the call option resulting from the modeling was evaluated using the fully implicit finite difference method and an algorithm – known in international literature as Projected

Successive Over Relaxation (Psor) – to solve the system of linear equations resulting from this method. 

The demand for complex financial instruments and the availability of powerful personal computers have made the direct numerical solution of the governing pricing equation an increasingly appealing approach to pricing. Therefore in this paper we chose to use the finite difference method for solving the differential equations that model the real option value instead of binomial or trinomial trees.

The three finite difference methods that have been commonly used to solve the differential equations that describe the behavior of option value are the explicit finite difference, Crank-Nicholson, and the fully implicit finite difference. The first two methods have stability problems, as will be discussed in more detail in this paper, hence the choice of the fully implicit finite difference method for this paper.

These methods, especially the fully implicit finite difference method, lead to the solution of systems of linear equations, as will be shown in this work. These systems of linear equations could be solved in various ways, for example, with a direct solution through the decomposition algorithm known as lower-upper (LU) where the matrix is decomposed into two sub-matrices: a lower triangular and an upper triangular. Alternatively they could be solved by interactive algorithms. In this paper, we chose to use an interactive algorithm based on the Psor, which – under a computational point of view – is more efficient than the direct solution.

The main objective of this paper is to analyze a forest investment in light of the theory of real options. In other words, this paper examines the value of a forest investment and the optimal harvesting time treating the forest as a real option, similar in nature to an American type call option, with the costs of harvesting as the exercise price of the option, given stochastic timber prices which behave as a geometric Brownian motion (GBM).

The theme is relevant both from an academic standpoint and from the aspect of practical application, given the importance of quantification of managerial flexibility in evaluating capital investment projects under uncertainty. Moreover, as far as the authors know, there are no other studies conducted in Brazil, which seek to determine the optimum time to cut a stand of trees, using a real options approach that have solved the differential equations using the numerical method and the interactive technique implemented in this work.

## REAL OPTIONS

When a company buys a machine, it is actually buying a stream of net revenues which will occur in future periods. To decide whether to buy the machine or not, the company has to calculate the present value of this stream discounting future cash flows at the appropriate rate. This is known as the net present value method which is the traditional approach to explain investment decisions (NICHOLSON, 2002).

The analysis of investment opportunities as options is the product of over a decade of research by several economists and is a topic still very active in academic publications. The traditional rules of the method of discounted cash flow ignore the irreversibility of investment and the options (or flexibility) that exist in a project such as, for example, the option to defer investment, to abandon, to increase or decrease the scale, to abandon temporarily and later to resume a project, and so on. So in a sense, techniques based on discounted cash flow also contradict the orthodox economic view of supply and demand, according to which the company enters the market and expands when the price exceeds the average cost of long-term and leave or contracts when the price falls below the average variable cost (DIXIT; PINDYCK, 1994).

Thus, many academics and business managers now recognize that the traditional approach for the evaluation of capital investment projects, i.e., the method of discounted cash flow, cannot adequately capture the flexibility of management to adapt and revise decisions in response to unexpected developments in market and the current flow of information (TRIGEORGIS, 2000).

When a firm makes an irreversible investment decision, it is exercising its option to invest and giving up the possibility of waiting until new information – which may affect the results – is available. This option value, which is commonly ignored by traditional approaches, should be included as part of the investment cost. The analysis of investment opportunities as real options allows the consideration of this flexibility (DIXIT; PINDYCK, 1994).

The term "real options" was first used by Myers (1977), which was the first author to identify that the assets in company could be viewed as an option. So, the real option theory has its origin in the theory of financial options, from which it inherits the mathematics and assessment techniques. However, given the specific characteristics of real options, different pricing mechanisms and simulation may be necessary.

## METHODOLOGY

Harvesting generates income from the sale of logs but it also involves costs such as harvesting costs, the loss of future income (net of administration costs) that could result from a standing forest, and the loss of additional timber volume resulting from letting the trees grow for another period. On the other hand, in a context of random prices if the harvest is postponed until the next period, the owner also faces uncertainty over prices that may be higher or lower than they are in the current period. Assuming that timber prices, P, follow some known stochastic process that is given in general terms by:

$$dP = a(P,t)dt + b(P,t)dz \tag{1}$$

where:

- a(P,t) is the drift term,
- b(P,t) it is the variance term.

The terms a(P, t) and b(P, t), are non stochastic known functions, and dz is the increment of a Wiener process. It is assumed that wood volume in a stand of trees, X, grows according to a known deterministic function, dependent of age: dX(t) = g(t)dt.

The decision to harvest follows the modeling proposed by Insley (2002). The decision can be specified as an optimal stopping problem that is solved using the technique of dynamic programming (DIXIT; PINDYCK, 1994). The income is designated by R and the wood volume by X. Therefore R = X.P where P follows the stochastic process shown in Equation (1). The Bellman equation is given below, where V(R, t) it is the value of the option of cutting the forest in the time t:

$$V(R,t) = \max\left[R(t)K; A(t)\Delta t + (1 + r\Delta t)^{-1} E(V(R + \Delta R, t + \Delta t))\right] \quad (2)$$

In Equation (2), R(t) is the income when the trees are cut in time t, K is the harvesting costs, A(t) it is an eventual additional income (liquid of administration costs) originating from the standing forest in period t, if the trees are not cut (this income can be generated, for instance, by amenity and for that matter, in this paper, this eventual additional income will be from now on called amenity). The instantaneous discount interest rate is given by r. Given certain regularity conditions (DIXIT; PINDYCK, 1994), for each t there will be a critical value of income, R *, it is optimal to preserve the forest if R < R *, while cutting is optimal if R > R *. The solution for the problem of cutting the trees involves discovering the free border, R = R * (t).

Following Dixit and Pindyck (1994) standard arguments, taking the limit as $\Delta t \to 0$, and applying Ito's Lemma, from Equation (2) we derive a partial differential equation satisfied by the value function in the continuation region:

$$rV(R,t) = A(t) + V_t + \left[a(R,t)X + \frac{g(t)}{X(t)}R(t)\right]V_R + \frac{1}{2}b^2(R,t)X^2V_{RR} \quad (3)$$

The optimal stopping problem (Equation 3) will now be re-specified in the form that is more useful to valuing an American-type call option with a free boundary. The problem is formulated as a linear complementarity problem (WILMOTT; DEWYNNE; HOWISON, 1993). There is a possible alternative formulation, the variational inequality, but both lead to the same result (INSLEY, 2002). Both formulations eliminate any explicit dependence on the free boundary; the free boundary can be recovered after the option problem valuation is solved (WILMOTT, 1998). 15

In order to formulate the tree harvesting problem as a linear complementarity problem, we write Equation (3), with τ defined as the remaining time, τ = T – t:

$$HV = rV(R,\tau) + V_{\tau} - \frac{1}{2} b^{2}(R,\tau) X^{2} V_{RR} - \left[ a(R,\tau) X + \frac{g(\tau)}{X(\tau)} R(\tau) \right] V_{R} - \mathrm{A} \quad (4)$$

The problem of the linear complementarity can now be specified as:

$$\begin{aligned} &(\mathrm{i})\, HV \geq 0 \\ &(\mathrm{ii})\, V(R,t)(R-\mathrm{K}) \geq 0 \\ &(\mathrm{iii})\, HV(V(R,t)(R-K)) = 0 \end{aligned} \quad (5)$$

This formulation can be seen intuitively as a description of the rational individual's strategy when holding an American option. Part (i) of the linear complementarity problem specifies that the required return (rV) less the actual return from delaying the harvest will not be negative. If HV = 0 then the required return from holding the option equals the actual return, and it is optimal to maintain the option. If HV > 0, then the required return exceeds the actual return, implying that the option should be exercised. The case when HV < 0 implies that the actual return exceeds the required return, a situation that one would not expect to persist in competitive markets. Part (ii) of the problem of linear complementarity establishes that the value of the option to harvest, V(R,t), can never go below the value of cutting the trees immediately, (R – K). This follows from the fact that the option to harvest can be exercised any time (American-type option). If the value of the option falls to the level of payout, it would be immediately exercised; therefore it would never fall below the payout value. Part (iii) establishes that either (i) or (ii) (or both) will hold as a strict equality. If HV =

0, then it is optimal to wait; if V(R,t) – (R – K) = 0, then it is optimal to cut. If both are identically null then the value of cutting is the same as the value of waiting and the owner would be theoretically indifferent to the two alternatives (INSLEY, 2002).

For a numerical solution of the system of inequalities (5) it is necessary to specify the boundary conditions. It is noticed that because the linear complementarity problem does not depend explicitly on the free boundary, it is not necessary to specify the value matching and smooth pasting conditions. Rather, these conditions are a consequence of this formulation (FRIEDMAN apud INSLEY, 2002).

Boundary condition #1: for nonzero values of X, as $R \rightarrow 0$, $P \rightarrow 0$, since R = X.P. From Equation (1), in order to avoid negative prices, it is necessary that $b \rightarrow 0$, when $P \rightarrow 0$ and $a \geq 0$ when $P \rightarrow 0$. Therefore when $R \rightarrow 0$ is possible to rewrite Equation (4) as:

$$HV = rV(R,\tau) + V_{\tau} - a(R,\tau)XV_{R} - \mathrm{A} \tag{6}$$

Being supposed that part (i) of the System (5) is a strict equality and the expression in part (ii) is strictly positive, this means that HV = 0 so that:

$$V_{\tau} = a(R,\tau)XV_{R} - rV(R,\tau) + \mathrm{A} \tag{7}$$

Equation (11) is a first order hyperbolic equation that has outgoing characteristics and therefore no further boundary conditions are required (HALL; PORSCHING, 1990). More precisely it can be shown that, since $b(R,t) \rightarrow 0$ faster than $R^{1/2}$, no boundary condition is necessary at R = 0.

If instead harvesting is optimal then part (ii) of system (5) will be equal to zero. The value of the option when $R \rightarrow 0$ is exactly equal to the payout, which will be the negative of harvesting costs (that is, V(R,t) = – K).

Boundary condition #2: as revenue, R, gets very large, the boundary condition that seems intuitively reasonable is given by:

$$V(R,\tau) = \gamma(\tau)R \tag{8}$$

for some function $\gamma(\tau)$. As R tends to the infinite, it remains little potential of ascent for the option, due to capital gains on the forest. It is assumed, therefore, that the value of the option is proportional to R. This implies that $V_R = \gamma(\tau)$ and $V_{RR} = 0$. Therefore:

$$HV \equiv rV(R,\tau) + V_\tau - \left[\frac{a(R,\tau)X}{R} + \frac{g(\tau)}{X(\tau)}\right]V(R,t) - A,_{R\to\infty} \qquad (9)$$

Even for a very large R, it may be optimal to postpone harvesting if the trees are still growing quickly. If we are in the continuation region where it is optimal to postpone harvesting, then H*V* = 0 and Equation (9) becomes:

$$V_\tau = V(R,\tau)\left[\frac{a(R,\tau)X}{R} + \frac{g(\tau)}{X} - r\right] + A,_{R\to\infty} \qquad (10)$$

If instead HV > 0, meaning that harvesting is optimal then part (ii) of System (5) is a strict equality, and ignoring K (because R is large) we have:

$$\begin{aligned} &V(R,\tau) - R = 0 \\ &\left(\gamma(\tau) - 1\right)R = 0, \quad \gamma(\tau) = 1 \end{aligned} \qquad (11)$$

Thus, as R increases, the function $\gamma$ assumes value = 1, if harvesting is optimal. In the numeric solution of the problem, a number arbitrarily large is chosen for the maximum value of R. It was checked to make sure that increasing the maximum R further does not change the results significantly.

Terminal condition: as the remaining time tends to zero, the value of the option is either the income from cutting, or the amenity value, whichever is larger:

$$V(R,\tau = 0) = \max[R - K, A] \qquad (12)$$

The numeric algorithm to determine the value of the option involves the discretization of the linear complementarity problem (System 5) using the fully implicit finite difference method (WILMOTT; DEWYNNE; HOWISON, 1993). The problem of linear complementarity was solved in this paper, at each time step, by the Psor method, which is a modification of the traditional SOR method (WILMOTT; DEWYNNE; HOWISON, 1995). The problem could also be solved by the penalty method (ZVAN; FORSTYTH; VETZAL, 1998) as was done by Insley (2002). To solve the system of linear equations that result from this method we implemented software written in C especially for this purpose.

## HYPOTHESES

The following hypotheses were formulated in this paper:

- $H_{O,1}$: prices follow a GBM.
- $H_{A,1}$: prices follow a mean reverting diffusion process.
- $H_{O}$: is implied or subtended for the next hypotheses.
- $H_{A,2}$: the value of the stand of trees varies as a function of wood price volatility.
- $H_{A,3}$: the value of the stand of trees varies as a function of interest rates.
- $H_{A,4}$: the value of the stand of trees varies as a function of harvesting costs.

## SAMPLE AND DATA

A historical series of wood prices is necessary to estimate the parameters of the model. In the specific case of this paper, it is necessary a historical series of prices of reforested eucalyptus, for use in cellulose production, to estimate the value of the drift rate (g) and the variance rate ($\sigma^2$).

To check whether prices revert to the mean, we used a unit root test known as Augmented Dickey Fuller (ADF). However, as Dixit and Pindyck (1994) alert, it is necessary a quite long time series, with data from many years, in order to determine with some degree of confidence whether the variable in fact reverts to the mean.

We obtained a Brazilian time series of industrially planted eucalyptus prices, for use in cellulose production, in the Bauru region of São Paulo, Brazil, published by the Centro de Estudos Avançados em Economia Aplicada (2008) of Escola Superior de Agricultura "Luiz de Queiroz" of the University of São Paulo. It is a monthly time series of prices beginning in October of 2002 and extending until August of 2007, covering a period of almost 5 years, but insufficient to be used to verify whether prices revert to the mean. The prices were deflated and converted to a constant Brazilian currency of August/2007, using two different price indexes. There were not significant differences among them. It is important to emphasize that due to periods of high inflation in Brazil, deflated prices could be very different depending on the price index that was used.

The model requires also an equation that supplies wood volume in eucalyptus stands in function of the age of the forest. The equation below was obtained in Rodriguez, Bueno and Rodrigues (1997).

$$V_t = 751{,}336 e^{6{,}0777 t^{1}} \tag{13}$$

where $V_t$ is the wood volume produced measured in m³/ha and $t$ is the age of the forest in years. The domain of this function is $2 \leq t < 30$ , therefore it was supposed that the forest does not produce marketable wood before 2 years and does not grow after the 30 years.

The exercise price of a call option is represented in the real option model by the harvesting cost. We estimated the harvesting cost in constant *Real* (R$) as of November/2007, with data obtained in *Agrianual – Anuário da Agricultura Brasileira* (2006, 2007), indexed by the price index IGP-DI (FGV, 2008) in the period from October/2002 to August/2007. The average harvesting cost in the period, in constant *Real*, was R$ 12.04/m³, which represents 19.34% of the average price of the wood in the period. Souza, Rezende and Oliveira (2001) mention that harvesting cost in Brazil is falling in proportion to the price of the wood, probably due to the mechanization of the crop: in the 1960's harvesting costs represented about 50% of the revenue obtained by the sale of the logs and now this cost represents less than 20%.

## DATA ANALYSIS

The model requires as input data, among others, the drift rate (g) and the variance rate ($\sigma^2$), which were obtained from the timber price time series. The historical prices were deflated, the natural logarithm of the deflated prices was obtained $(p_t = 1nP_t$ and the average of the logarithm of the prices was obtained by:

$$\overline{p_t} = \frac{\sum_{t=2}^{n} p_t \, p_{t-1}}{n} \quad (14)$$

If it is assumed that price follows a process of GBM, the maximum-likelihood estimates of the variance rate, $\sigma^2$, will be $s^2$, where s is the standard deviation of the time series $p_t - p_{t-1}$, and the estimates of maximum-likelihood of the drift rate g is obtained by the Equation (15) below after making the necessary adjustments to convert the estimates from a monthly to an annual basis:

$$\mu = m + \frac{1}{2} s^2 \quad (15)$$

where m is the average and s is the standard deviation of the series.

Estimates of the annual risk free interest rate, capitalized continually (r), will also be necessary. In this paper **r** was adopted as 10% a year.

It is also necessary to obtain the quotient of two equations $\frac{g(t)}{X}$. The equation X that appears in the denominator is Equation (13) that estimates timber volume as a function of age, already presented above. The equation g($\tau$) is the first derivative of the timber volume Equation (13), given below:

$$g(\tau) \equiv \frac{dV}{dt} = 4566.3948 t^{-2} e^{\frac{-6.0777}{t}} \quad (16)$$

It can be verified that the timber volume growth rate, according to this model, is negligible after the 30 years. Due to the lack of information on the value A, which represents other possible incomes, liquid of costs, which could occur for the standing forest, the value of A was considered zero, i.e., there is no other revenue while the forest is not harvested.

The parameters presented above were used as input data to the software, in order to model the value of the stand of trees and the harvesting decision. The value of the option, which results from the model, represents the market value of the standing trees in the several ages. Table 1 below display the market values of the stand of Brazilian eucalyptus, with seven, twelve, seventeen and twenty-two years of age, for four different prices of wood, when the price diffusion model is GBM.

TABLE 1 – VALUE OF THE STAND OF EUCALYPTUS IN BRAZIL.

| Wood price | Option value | | | |
|---|---|---|---|---|
| | 7 years | 12 years | 17 years | 22 years |
| R$ 122.00 | R$ 46,616 | R$ 49,783 | R$ 57,779 | R$ 62,670 |
| R$ 95.50 | R$ 35,922 | R$ 37,786 | R$ 43,856 | R$ 47,568 |
| R$ 69.00 | R$ 25,040 | R$ 25,794 | R$ 29,932 | R$ 32,466 |
| R$ 42.50 | R$ 14,158 | R$ 14,846 | R$ 16,009 | R$ 17,364 |

Source – Produced by author.

The optimal harvesting time depends on the current wood price which was considered to be stochastic following a GBM. Critical prices are the prices for which it is indifferent to exercise the option or not. According to our model to exercise the option means to harvest the stand of trees. In this context a critical price is the minimum wood price at each age of the forest which would make it convenient to cut the trees and sell the logs. Table 2 below displays the critical prices for several ages of the stand of trees. It can easily be noticed that the critical prices fall dramatically after 7 years of age, indicating that the trees are most likely to be cut after that age.

TABLE 2 – CRITICAL PRICES.

| Age in years | Critical prices R$ |
|---|---|
| 2 | R$ 238 |
| 7 | R$ 203 |
| 12 | R$ 75 |
| 17 | R$ 19 |
| 22 | R$ 16 |

Source – Produced by author.

The hypothesis $H_{0,1}$ of this paper states that the prices follow a GBM and its alternative hypothesis $H_{A,1}$ states that the prices follow a diffusion process with mean reversion. So, initially, the price series was subjected to a unit root test known as ADF, as suggested by Dixit and Pindyck (1994). The aim of the unit root test is to detect stationarity in time series. The null hypothesis could not be rejected. This means that prices are likely to behave as a random walk. The price series, however, is too short for us to affirm that safely.

Hypothesis $H_{A,2}$ of this paper stipulates that the value of the forest stand varies as a function of wood price volatility. A sensitivity analysis of the model to the variation of volatility was conducted in order to examine this hypothesis. The value of volatility ($\sigma$) which was originally $\sigma = 0.10072$ was tripled to σ = 0.3022. Table 3 below shows the effect of this increase in volatility on option value, in other words on the value of the forest. 23

TABLE 3 – OPTION VALUES (R$) X VOLATILITY (Σ = 0.10 AND Σ = 0.30).

| Age in years | Volatility | Wood prices R$ | | | |
|---|---|---|---|---|---|
| | | 42.50 | 69.00 | 95.50 | 122.00 |
| 7 | σ = 0.10 | 13,846 | 24,532 | 34,060 | 42,439 |
| | σ = 0.30 | 13,930 | 25,040 | 35,922 | 46,616 |
| 12 | σ = 0.10 | 14,158 | 25,794 | 37,786 | 49,783 |
| | σ = 0.30 | 14,248 | 25,796 | 37,786 | 49,783 |
| 17 | σ = 0.10 | 16,009 | 29,932 | 43,856 | 57,779 |
| | σ = 0.30 | 16,009 | 29,932 | 43,856 | 57,779 |
| 22 | σ = 0.10 | 17,364 | 32,466 | 47,568 | 62,670 |
| | σ = 0.30 | 17,364 | 32,466 | 47,568 | 62,670 |

Source – Produced by author.

It can be observed that the influence of a volatility increase is greater in younger ages. That is, the value of the forest is more sensitive to variations in volatility at younger ages. This result was expected because the model sees the forest as an option and it is natural that the volatility has more influence when the option has a long lifetime to maturity. The hypothesis $H_{A,2}$ was confirmed.

The hypothesis $H_{A,3}$ stipulates that the value of the forest stand changes as a function of interest rates. We conducted two sensitivity analysis changing the original interest rate – which was 10% per year – first to 5% per year and then to 15% per year. The results are presented in Table 4 below and show significant differences between the values of the option.

TABLE 4 – OPTION VALUE (R$) – FUNCTION OF WOOD PRICES AND INTEREST RATES.

| Age in years | Interest rates per year | Wood prices (R$) | | | |
|---|---|---|---|---|---|
| | | 42.50 | 69.00 | 95.50 | 122.00 |
| 7 | 5% | 14,595 | 29,087 | 42,112 | 50,126 |
| | 10% | 13,846 | 24,532 | 34,060 | 42,439 |
| | 15% | 11,621 | 23,790 | 37,796 | 49,393 |
| 12 | 5% | 15,092 | 29,689 | 43,093 | 55,126 |
| | 10% | 14,158 | 25,794 | 37,786 | 49,783 |
| | 15% | 13,793 | 25,796 | 37,786 | 49,783 |
| 17 | 5% | 16,157 | 30,003 | 43,897 | 57,805 |
| | 10% | 16,009 | 29,932 | 43,856 | 57,779 |
| | 15% | 16,009 | 29,932 | 43,856 | 57,779 |
| 22 | 5% | 17,364 | 32,466 | 47,568 | 62,670 |
| | 10% | 17,364 | 32,466 | 47,568 | 62,670 |
| | 15% | 17,364 | 32,466 | 47,568 | 62,670 |

Source – Produced by author.

It can be seen that although the value of the forest is a function of the interest rates, it is far more sensitive to variations in interest rates at younger ages than at older ages. Hypothesis $H_{A,3}$ was also confirmed.

Hypothesis $H_{A,4}$ of this paper states that the value of the forest stand changes as a function of harvest costs. Two sensitivity analyses were conducted to verify this hypothesis. The original harvesting cost is R$ 12.04. The first sensitivity analysis used zero as harvesting cost, i.e., there are no costs to harvest the trees. The second analysis used a harvesting cost of R$ 36.00, nearly triple the original cost. The results are presented in Table 5 below and show significant differences between the values of the option. In all cases, lower costs produce higher values of the forest, which is intuitive. This means that hypothesis $H_{A,4}$ was also confirmed.

TABLE 5 – OPTION VALUE (R$) X WOOD PRICES AND HARVESTING COSTS.

| Age in years | Harvestig costs (R$) | Wood prices (R$) | | | |
|---|---|---|---|---|---|
| | | 42.50 | 69.00 | 95.50 | 122.00 |
| 7 | 0.00 | 17,504 | 28,417 | 39,314 | 50,036 |
| | 12.04 | 13,846 | 24,532 | 34,060 | 42,439 |
| | 36.00 | 8,244 | 18,650 | 29,381 | 39,954 |
| 12 | 0.00 | 19,244 | 31,241 | 43,237 | 55,234 |
| | 12.04 | 14,158 | 25,794 | 37,786 | 49,783 |
| | 36.00 | 5,301 | 15,714 | 27,293 | 39,116 |
| 17 | 0.00 | 22,336 | 36,259 | 50,183 | 64,106 |
| | 12.04 | 16,009 | 29,932 | 43,856 | 57,779 |
| | 36.00 | 4,598 | 17,341 | 31,265 | 45,188 |

(continue)

TABLE 5 – OPTION VALUE (R$) X WOOD PRICES AND HARVESTING COSTS (continuation).

| Age in years | Harvestig costs (R$) | Wood prices (R$) | | | |
|---|---|---|---|---|---|
| | | 42.50 | 69.00 | 95.50 | 122.00 |
| 22 | 0.00 | 24,226 | 39,328 | 54,43 | 69,532 |
| | 12.04 | 17,364 | 32,466 | 47,568 | 62,670 |
| | 36.00 | 4,144 | 18,809 | 33,911 | 49,013 |

Source – Produced by author.

## FINAL CONSIDERATIONS

The problem of harvesting a stand of trees was treated in this work as a real option, similar in nature to an American call option, with the costs of harvesting trees as exercise price.

It is known that there is no closed analytical solution for the problem of evaluating an American call option, with exercise price different from zero, on an underlying asset that pays dividends. Then, when a forest investment is evaluated as a real option, it is necessary to numerically solve the differential inequalities of the linear complementarity problem. This paper used the fully implicit finite difference method, with an interactive algorithm for the solution of the resultant linear system of simultaneous equations, denominated Psor. It is a robust technique to evaluate real options of this type, as it was demonstrated.

The sensitivity of the option value – which is the dependent variable of the problem – to some of the independent variables was also examined. In all cases these independent variables showed significant influence on the option value.

Table 6 summarizes the assumptions made above and indicates their acceptance or rejection. The hypothesis $H_{A,1}$ – the alternative to $H_{O,1}$ – stipulating that the price movement follows a mean reverting process, although interesting from a theoretical standpoint, cannot be accepted. The other hypotheses were accepted.

The main objective of this paper – which was to examine a forest investment in light of the theory of real options by analyzing the value of the investment and the optimal harvesting decision – has been reached. The forestry investment – considering wood prices to be stochastic and to follow a GBM – was treated as a real option, similar in nature to an American type call option, with harvesting costs as the option exercise price. It was possible to obtain a numerical solution to the problem of pricing the real option investigated in the paper using the fully implicit finite difference method and the Psor algorithm. In a context where the aspects of sustainability are growing in importance identifying the optimum harvesting time enables the productivity of forests to be maximized, which in turn should decrease the environmental impact.

TABLE 6 – SUMMARY OF ACCEPTANCE/REJECTION OF HYPOTHESES.

| Hypothesis | Description | Status |
|---|---|---|
| $H_{0,1}$ | prices follow a GBM | Accepted |
| $H_{A,1}$ | prices follow a mean reverting diffusion process | Rejected |
| $H_{A,2}$ | the value of the stand of trees varies as a function of wood price volatility | Accepted |
| $H_{A,3}$ | the value of the stand of trees varies as a function of interest rates | Accepted |
| $H_{A,4}$ | the value of the stand of trees varies as a function of harvesting costs | Accepted |

Source – Produced by author.

# Competitividade da indústria brasileira de celulose: análise de um investimento florestal com opções reais


## RESUMO

O Brasil enfrenta desafios que precisam ser superados para que ele possa se transformar em um competidor produtivo e importante no mundo. O conjunto desses desafios ficou conhecido como "Custo Brasil". Mas, apesar disso, alguns setores da economia brasileira são tão produtivos que nem mesmo esse custo mais alto pode prejudicar a sua competitividade mundial. A indústria de celulose é um deles. Devido à vantagem comparativa do país na produção de celulose de fibra curta e a importância do setor para a economia brasileira, o presente trabalho analisa um projeto de investimento florestal. Optou-se por usar uma abordagem de opções reais para avaliar o investimento florestal e o tempo ideal para a colheita, uma vez que esta abordagem consegue capturar o valor da flexibilidade da decisão de investimento. A opção que resulta dessa modelagem não tem solução analítica fechada e por isso recorreu-se a uma solução numérica, conhecida como diferenças finitas totalmente implícitas, que leva a um sistema de equações lineares, que foi resolvido com um algoritmo iterativo chamado Projected Successive Over Relaxation (PSOR), implementado computacionalmente por meio de um software desenvolvido especificamente para esse fim. O objetivo principal deste trabalho, que era de examinar um investimento florestal à luz da teoria das opções reais, foi alcançado e identificado o tempo de colheita ideal. Em um contexto em que os aspectos da sustentabilidade estão crescendo em importância, identificar o momento ideal da colheita permite que a produtividade das florestas seja maximizada, o que consequentemente deve diminuir o impacto ambiental da indústria.

## PALAVRAS-CHAVE

Avaliação de investimento florestal. Indústria brasileira de celulose. Opções reais. Método de diferenças finitas totalmente implícitas. Algoritmo Psor.

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