# From heterogeneous expectations to exchange rate dynamics

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## ABSTRACT

The purpose of this paper is to analyze how heterogeneous behaviors of agents influence the exchange rate dynamic in the short and long term. We examine how agents use information and what kind of information, to make decisions to anticipate the exchange rate. We investigate methodology based on interactive agent simulations, following the Santa Fe Artificial Stock Market. Each trader is modeled as an autonomous, interactive agent and the aggregation of their behavior results in foreign exchange market dynamics. A genetic algorithm is used to model agents behavior, and the simulated market tends to replicate the real EUR/USD exchange rate market. We consider four kinds of agents with pure behaviors: fundamentalists, positive and negative feedback traders, naive traders and news traders (positive and negative). To reproduce stylized facts of the exchange-rate dynamic, we find it necessary to pick a correct proportion for each agent type, eschewing need for mimetic behavior, adaptive agents or pure noisy agents.

## KEYWORDS

Exchange rate dynamic. Heterogeneous expectations. Interactive agent behavior. Genetic algorithm. Learning process.

## INTRODUCTION

Agent heterogeneity is obvious from examining the large number of studies using survey data. Such data are appealing since they allow us to measure exchange rate expectations directly. Using a panel data of biweekly surveys on the yen-dollar exchange rate expectations for forty-four Japanese institutions from May 1985 to June 1987, Ito (1990) finds a wide dispersion in individual expectations. In the three-month horizon, he observes that one extreme predicted a 3.25 percent depreciation of the yen, while the other extreme predicted a 4.76 percent of appreciation. Mac Donald (1992), which compares the exchange expectations in the G7 countries from October 1989 to March 1991, confirms this result. Frankel and Froot (1986a, 1986b, 1987a, 1987b) also show that the standard deviations of the expectations' means increase considerably when the horizon decreases. Ito (1990) and Takagi (1991) find significant individual effects in participants' expectation formation. Those individual effects have characteristics of wishful expectations: exporters expect yen depreciation and importers expect a yen appreciation. On the whole, survey data show that for short-term predictions (from one week to one month), respondents tend to forecast by extrapolating recent trends (extrapolative expectations), while for the long term (six to twelve months), they tend to forecast a return to long-run equilibrium such as PPP (regressive expectations).

This durable combination of short-term extrapolative expectations with destabilizing effects, and long term regressive ones that stabilize, highlights two conflicting ideas of the exchange dealers: long term responses seem to express the operators' economic understanding, the fundamentals. Short-term responses seem to correspond to the market logic and reveal the true opinion, at that time, of the respondent. The observation of past trends, the use of chartist methods, and the goal of being in the market, thus neglecting the long term, seem to prevail. Interestingly, it is especially in the short run that specialists' trading takes place. The tremendous volume of foreign exchange trading is another piece of evidence that reinforces the idea of heterogeneous expectations since it takes differences among market participants to explain why they trade[1]. Goldstein et al. (1993) estimates that each customer's transaction generates, on average, four to five inter-dealer transactions in response to the price discovery operations they imply and that the speculative operators adopt.

Standard models of exchange rate determination, by contrast, assume the existence of identical investors who share rational expectations of future exchange rates, and who instantaneously and rationally discount all market information concerning this rate. While this assumption is crucial to allow a simple aggregation of common individual behaviors, it is not ideally suitable because it raises the question of the existence of transactions if all agents are strictly identical (ARROW, 1986). It follows that trading volume is low or zero, and that trading volume and price volatility are not serially correlated in any way. However, foreign exchange markets, as well as other financial markets, are characterized by striking time-series features: unit roots in level, together with fat tails in returns, and volatility clustering. As a result, there has been a search for alternative explanations for such realities.

Microstructure analyses investigate the role of the market's organizational or institutional characteristics on exchange rate determination: auction types, centralization of order flows, quotation rules, and the like. They see trading as a process of information transmission and market opinion discovery (LYONS, 1995). Exchange rate determination is therefore endogenous to the inter-bank market (VARIAN, 1989; PERRAUDIN; VITALE, 1996). To date, these studies have not answered the question of foreign exchange rate determination but have focused on the spread determination, the relation between trading volume and volatility, the marketplace transmission of volatility (LYONS, 1995; FLOOD, 1991, 1994; GOODHART; PAYNE, 1996; GOODHART; ITO; PAYNE, 1996; GOODHART; CHANG; PAYNE, 1997; EVANS; LYONS, 1999; OSLER, 2001). They all emphasize the role of traders as market-makers.

1 The banks in the BIS (1998) census reported that 81 percents of the spot trading, which represents a daily average of $ 1,5 trillions, takes place among the banks and other financial institutions, rather than with customers such as exporters and importers.

This paper relies on the growing literature on computational agent-based models. In a recent survey, LeBaron (2000) argues that computational agent-based models stress interactions and the dynamics of groups of traders learning about the relations between prices and market information. Under heterogeneity, expectations have a recursive character: agents must form their expectations from their anticipations of other agents' expectations, and this self-reference precludes expectations being formed by deductive means. Agents therefore continually form individual, hypothetical, expectational models, test these, and trade on the ones that predict best. Prices are driven endogenously by these induced expectations. Agents' expectations co-evolve in a world they co-create (ARTHUR et al., 1996). Arifovic (1996) considers a dynamic version of the Kareken and Wallace (1981) model of exchange rate formation in a two country overlapping generations world. Using a standard genetic algorithm procedure to update agents' decision rules, the simulations give exchange rate series which do not converge to any equilibrium. As it is outlined in LeBaron (2000), this result is related to the structure of the indeterminacy of the model. Arifovic and Gencay (2000) find that the model's equilibrium dynamic is not constant but exhibits bounded oscillations. Their time series analysis of the data indicates that the dynamics of exchange rate returns is chaotic. Lux and Schornstein (2005), using the same model, find that for particular parameterizations, the characteristics of exchange rate dynamics are very similar to those of empirical data (unit root in levels together with fat tails in returns and volatility clustering)[2]. However, they show that whether realistic time series characteristics materialize depends essentially on the mutation probability and the number of agents. This later finding casts doubts on the potential applicability of this model to real markets such as the foreign exchange market.

Our artificial market is initially inspired by the Santa Fe Stock Market, which is outlined in detail in Arthur et al. (1996) and LeBaron, Arthur and Palmer (1999), and is an extension of Neuberg and Bertels (2003) model.

2 Similar results for other markets are found by Lux and Marchesi (1999, 2000), Chen, Lux and Marchesi (2001), Kirman and Teyssiere (2001) among others.

However, it differs in some points. First, instead of a stock market, we simulate the USD/EUR exchange rate dynamic. Therefore, agent types are quite different. Secondly, as our goal is to reproduce actual exchange rate series, we use real data to create agents' decision rules.

Our analysis proceeds in the following steps: in section "Description of the market, we explain the different financial agents and their behaviors, then the learning process and expectation formation, market clearing and price formation, and finally the information set; the section "Simulation results" presents the simulation results of the exchange rate dynamic and the statistical properties of the simulated series.

## DESCRIPTION OF THE MARKET

**THE FINANCIAL AGENTS** We distinguish between different kinds of traders on the market, each having his own rationals and knowledge. Like any trader, the agent must be able to evaluate an action and form an expectation with respect to its future price. In this paper, we introduce four different types of behavior, which are described below.

- *Fundamentalists*: forecast a return to a long-run equilibrium and therefore have regressive expectations. They place buy (sell) orders if the current exchange rate is under (over) this fundamental value. In this paper, the fundamental value is determined according to uncovered interest rate parity.
- *Noise traders*: base their position, generally speaking, on feelings not justified by existing information. De Long et al. (1990a) and Shleifer and Summers (1990) consider pure noise traders who act randomly. We do not take account for this kind of trader because the foreign exchange market is a specialist one. Nevertheless, we introduce two kinds of noise traders traditionally described in literature (CUTLER; POTERBA; SUMMERS, 1990; DE LONG et al., 1990b). Positive feedback traders are those who buy when prices rise and sell when prices fall. Many forms of common behavior in financial markets can be described as positive feedback trading. It can result from extrapolative expectations about prices or

trend chasing. It can also result from stop-loss orders, which effectively prompt selling in response to price declines. A similar form of positive feedback trading is the liquidation of the positions of investors unable to meet margin calls. Negative feedback traders react negatively to previous price movements: they buy when prices fall and sell when prices rise.

- *News traders*: act in response to exogenous information that can be decomposed into three types: good news, bad news, and no news. As for noise traders, we distinguish between positive news traders and negative ones. The former react positively to the news (they buy when the news is good and sell otherwise), and the latter react negatively (they sell when the news is good and buy otherwise).
- *Naive traders*: they expect the exchange rate to remain stable (ARTHUS, 1992).

We consider only agents exhibiting pure behavior without re-learning.

THE LEARNING PROCESS OF AGENTS AND EXPECTATION FORMATION Some methods allow us to build agent-behavior models without formalizations using explicit equations. This is the case for genetic algorithms (GA), a method initially developed by Holland (1975) to study an adaptive system. They are now applied to the study of learning systems and can be used to model the natural adaptive learning process to allow artificial systems to be built using the natural mechanisms of the learning process. Their simplicity explains their success. GAs allow us to simulate inductive agents' learning in a dynamic environment. Agents learn rules, called classifier system, that replicate the decision making process. These rules are built and corrected with the information that the agents have on the environment. The learning process allows the agents to form hypotheses and to formulate expectations about the market.

In our model, each agent needs to be able to decide whether he wants to buy or sell a particular currency and at what price. He therefore needs to have decision rules that allow him to formulate some kind of expectation as to the future evolution of the exchange rate. He does so on the basis of information at his disposal. In our model, we have chosen to implement a classifier system in wich different decision rules are represented as if-then rules. At a given moment, if a condition of his set satisfies the present situation in the environment, the agent will take the corresponding action. The condition of each rule is a chain of characters ("0", "1" or "#") determining whether the rule is equivalent to the market situation. This equivalence is achieved if the characters along the chain of the condition are similar to the characters along the chain of the market situation. In the case of character "#", there is always an equivalence to the extent that it expresses the indifference between the characters "1" and "0". As for the action, it is a chain of characters representing the value of two parameters *a* and *b* in binary fashion. These parameters allow us to compute the expected future exchange rate in the following way:

$$E\left[e_{t+1}\right]=a\left(e_t\right)+b \quad (1)$$

where $e_t$ is the exchange rate at time t. For each agent, a set of rules allowing the calculation of these expected prices is generated using genetic algorithms. Initially 2000 rules are generated and during the learning process, depending on the agent's type, this number is reduced. In contrast to previous studies, the learning process is based on real data. Risk aversion is expressed in terms of the CARA utility function that, for the sake of comparability, is taken from (1).

$$U(w)=-e^{-\lambda w} \quad (2)$$

where *w* represents the wealth of the trader and l indicates the degree of risk aversion.

Some of the original 2000 rules may be more efficient than others. Those yielding more accurate expected prices, and therefore a higher financial gain, have a higher reproduction rate and a higher probability of survival. Before starting trading simulations, each agent learns the market dynamic passively. This means that he uses real market data to construct his set of rules. The effectiveness of the decision rules is defined in relation to the error generated by the rule and is computed as follows:

$$Error(rule)=\left(E\left[e_{t+1}\right]-e_{t+1}\right)^2 \tag{3}$$

Using (1) results in

$$Error(rule)=\left(a\left(e_t\right)+b-e_{t+1}\right)^2 \tag{4}$$

A perfect rule computes an expected value equal to the exchange rate and the error is null. This kind of rule has a maximum evaluation value, C. We then obtain a rule evaluation function, also called the strength of the rule, defined as:

$$Eval(rule)=C-\left(a\left(e_t\right)+b-e_{t+1}\right)^2 \tag{5}$$

A process that assumes agents' rationale completes this evaluation function. Each rule must reflect the specific agent type. Let us take an example: the rule of a fundamentalist agent receives a better evaluation function if this rule leads to a value of exchange rate that is based on interest rate parity. Otherwise, this rule will be under-evaluated, even if the predicted value is not so bad.

**MARKET CLEARING AND PRICE FORMATION** Intersecting orders to buy and sell create the dynamics of exchange rates. The market clearing mechanism is similar to that of the Santa Fe Stock Market in which bids are continuously resubmitted until a price (an exchange rate in our case) is formed that clears the market. For each period of time, the agents try to optimize the allocation of risky and non-risky assets, i.e., US currency versus Euro currency. Initially, the exchange rate previsions made by agent *i* at time *t* are normally distributed with an average of $E_{i,t}[e_{t+1}]$ and a variance $\sigma^2_{i,t,e}$. Demand (or supply) by agent *i* at time *t* is given by 

$$x_{i,t} = \frac{E_{i,t}(e_{t+1} - (1 + r_t)e_t)}{\lambda\sigma^2_{i,t,e}} \tag{6}$$

where $r_t$ is the interest rate of the non-risky asset at time *t*.

As the market is composed of *n* agents, we therefore have *n* equations with *n* + 1 unknown values: the *n* quantities of risky assets allocated by agents ($x_{i,t}$) and the exchange rate at time *t*. To close the system, we add an equation stipulating that total demand be equal to the number of available goods on the market: $\sum_{i=1}^{n} x_{i,t} = n$. At each time *t*, the exchange rate $e_t$ can be computed by resolving the $(n+1)*(n+1)$ system of equations.

**INFORMATION SET** In our model, information arrives at the market at regular intervals of time. Every iteration represents one day. The information is used differently depending on the agent type. As we previously defined agents, they use part of the available information. Information is composed of three parts: technical, fundamental and exogenous.

The technical part is provided by a two bit binary chain. The first character of the chain concerns the trends of the exchange rate. If the return of the exchange rate on the previous period is positive, then the value will be '1', and otherwise '0'. The second character reflects the absolute value of the trends. If the absolute value of the return is greater than 5 percent, then the value is '1', and '0' if lower.

The fundamental part of the information represents interest rate parity. The first character is constructed with a three month interest rate and a three month forward exchange rate. If the value of the forward exchange rate is lower than the computed exchange rate, the value of the character will be '1', '0' otherwise. The same reasoning is used at a six month horizon.

The third part of information is related to exogenous information. This information may vary from negative (-1) over neutral () to positive (1). We filter important events influencing the trends of the market. This kind of information would be, for instance, a terrorist event as well as an intervention of a Central Bank on interest rates.

## SIMULATION RESULTS

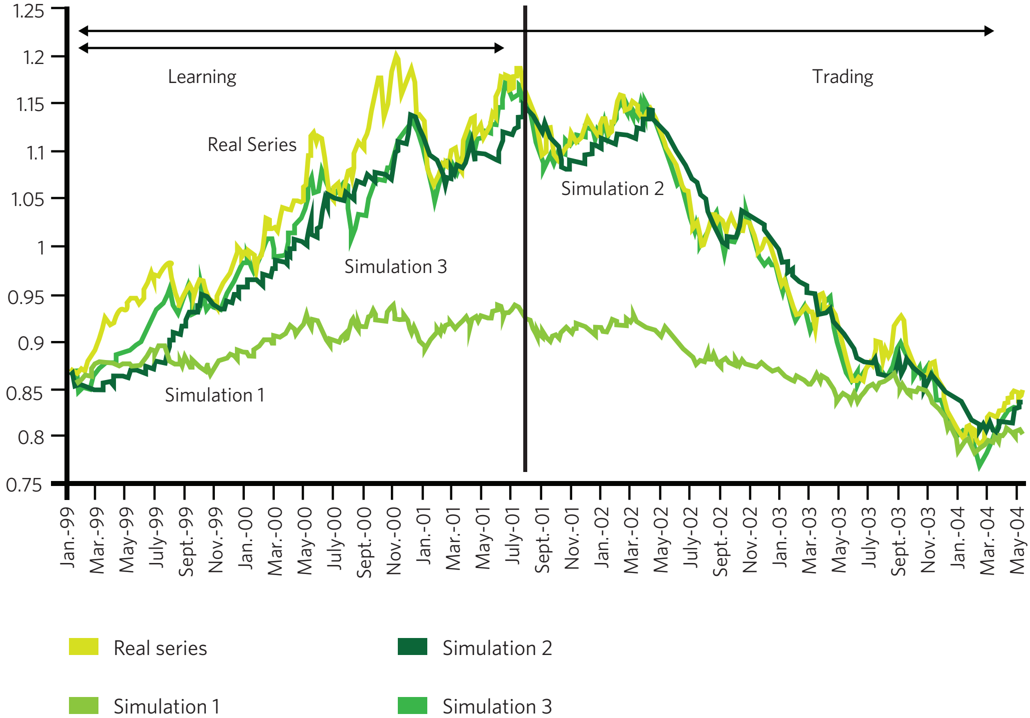
It is important to emphasize that we are focusing on understanding the dynamics of the exchange market and not on prediction. This model is validated in this respect if statistical properties can be compared to the real exchange rate values. The framework of simulations is to find the best ratio between the number of different agent' types in order to imitate the real market.

**PRESENTATION OF SIMULATIONS** We started our investigation with the creation of an individual agent' model, using real data on the EUR/USD exchange rate from January 1999. On a daily basis, 700 periods of time are considered for the learning process (until June 2001). Five different agents are created for each type. During this learning process, no market simulations are performed. Agents learn by trading like passive actors on the market. After this first step, simulations are performed from January 1999 to May 2004. During this period agents trade actively in the artificial market. Thus half the simulation period is outside the learning period. This avoids over-learning leading to pure reproduction, at least during the second part of the simulation period.

We present only the five most representative simulations that lead to the most reliable exchange rate dynamic.

- *Simulation 1*: We start our investigation with a purely fundamentalists' market. The simulations include three fundamentalists and no other agents. As can be seen in Graph 1, the simulated series exhibit a long-run equilibrium with a very low volatility compared to the actual exchange rate. Interestingly, over the last 12 months, the two series seem to converge, which may indicate that the market is driven by fundamentalists' beliefs.

GRAPH 1 – RESULTS OF SIMULATIONS 1 TO 3.




Source – Produced by authors.



- *Simulation 2*: A purely technical market excluding fundamentalist is explored next. Two positive feedback traders and two negative feedback traders are present in this market. We observe in this simulation a global shape that is closer to reality than the previous one. Nevertheless, the main characteristics of this market are two trends: one upward, one downward. The volatility is too low to be compared to reality. One could conclude that the global shape of the real exchange market is driven by technical forces. Unfortunately, nothing can explain the change of trends if we consider just technical traders. In this simulation, this could be due to the fact that technical agents, during learning, also had constructed a small number of fundamental rules.
- *Simulation 3*: A market with two positive feedback traders, two negative feedback traders and four fundamentalist agents seems to give more volatility and a more accurate dynamic. In this market, we observe the same global shape as reality and more frequent dynamic patterns resembling the real world.

GRAPH 2 – RESULTS OF SIMULATIONS 4 TO 5.


Source – Produced by authors.



- *Simulation 4*: We then simulate a market incorporating every type of agent. Two positive feedback traders, two negative feedback traders, two fundamentalists, two news agents, two negative news agents as well as two naïve agents trade in this market. We observe two main parts in the simulation. The first is a quite linear increase in the market and the second is a dynamic that is closer to reality. This ratio of agent types is more appropriate during a bear market than in a bull market. Therefore, we must find a better ratio able to explain every kind of dynamic.
- *Simulation 5:* The most appropriate ratio of agent types that we find is three positive feedback traders, one negative feedback trader, five fundamentalists, two positive news agents, one negative news and one naïve agent. This model seems to produce the most realistic market behavior.

STATISTICAL PROPERTIES To see whether our simulated series have realistic time-series properties, we use a battery of statistical tests. The results are detailed in appendix. First, we calculate the first four moments of the exchange rate returns' distribution and test for normality (Table 1 – see Appendix). Second, we analyze the auto-correlation of returns (Table 2 – see Appendix). Third, since previous studies have shown that (log) exchange rates series are non-stationary processes, we perform typical tests (Phillips & Perron and KPSS tests with constant and trends) for the presence of a unit root in our series (Table 3 – see Appendix). Finally, we compute the BDS tests to test the null hypothesis of i.i.d. returns and estimate the entropy (Table 4 – see Appendix).

As expected, the Bera-Jarque test for normality leads to a strong rejection of the null hypothesis for all series. We observe significant excess kurtosis in real and simulated return series, which confirms the existence of fat tails. There is no asymmetry in the real series and the fifth simulated series, which we consider to be the most representative. However, the variance of the simulated series are smaller than the real data. Concerning the Ljung-Box statistic, we conclude that there is a highly significant auto-correlation in level, squared, and absolute values in all simulated series, but not in the real exchange rates. The numerical precision used in the code of market models is such that the returns produced have a limited number of significant decimals. As a consequence, there is a limited number of different figures and therefore a higher auto-correlation of these series. Perhaps, another explication may be the codification of exogenous information in our market ("-1", "0", "+1").

As expected, unit root tests could not reject the null hypothesis of a unit root in the exchange rates or in the simulated series, with the exception of simulation 1. The different series are non-stationary processes.

In addition, we calculate the BDS statistic of Brock, Dechert and Scheinkman. The BDS statistic tests the null hypothesis of an identical and independent distribution and it is shown to be powerful against nonlinear alternatives. It is distributed asymptotically standard normal. For certain cases (especially when s/e is large) we reject the null hypothesis that the series is i.i.d., i.e., that the series is chaotic. We compute the entropy, which is the sum of the positive Lyapunov exponents. It provide a quantitative measure of the non-predictability of the chaotic system. All values are positive. We conclude that the predictability of the series is low. In other words, the lower the entropy, the higher is the potentiality of predictability, even if the system remains non-predictable in the long-run. In our case, we conclude that the exchange rate is more predictable in the short term but is difficult to predict in the long term.

## CONCLUSION

The purpose of this paper is to analyze how heterogeneous behaviors of agents influence the exchange rate dynamic in the short and long term. We examine how agents use information and what kind of information they use to make their decisions and form an expectation of the exchange rate. We investigate a methodology based on interactive agent simulations to reproduce the exchange rate dynamic of the EUR/USD exchange rate. To reproduce stylized facts of the exchange rate dynamic, we find it is necessary to pick a correct proportion for each agent type, eschewing need for mimetic behavior, adaptive agents, or purely noisy agents.

The next step is to incorporate more agents of each type to refine the best ratio of agent types. We can easily multiply the number of agents by ten leading to a hundred. We could also increase the amount of data. We can also improve the treatment of the exogenous information.

An interesting avenue of research would follow the study in terms of the portfolio value of agents, i.e., the wealth evolution of each agent, as well as a dynamic evolution of their risk aversion corresponding to their loss or gain.

# Das expectativas heterogêneas para a dinâmica da taxa de câmbio



## RESUMO

O objetivo deste trabalho foi analisar como o comportamento heterogêneo dos agentes influencia a dinâmica da taxa de câmbio em curto e longo prazos. Examinaram-se os tipos de informação que os agentes utilizam para tomar decisões que antecipem a taxa de câmbio. Foram investigadas metodologias baseadas em simulações de agentes interativos, seguindo o Santa Fe Artificial Stock Market. Cada *trader* foi modelado como um agente autônomo e interativo, e a agregação do seu comportamento resultou na dinâmica do mercado de câmbio. Um algoritmo genético foi usado para modelar o comportamento do agente, e o mercado simulado tende a replicar o mercado real de câmbio de euro/dólar. Foram considerados quatro tipos de agente com comportamento puro: fundamentalista, *trader* com *feedback* positivo e negativo, *trader* ingênuo e *trader* que segue notícias (positivas e negativas). Para reproduzir os fatos estilizados da dinâmica da taxa de câmbio, selecionou-se uma proporção ajustada para cada tipo de agente, a fim de evitar a necessidade de considerar comportamento mimético, agentes adaptativos ou agentes causadores de ruído puros.

## PALAVRAS-CHAVE

Dinâmica da taxa de câmbio. Expectativas heterogêneas. Comportamento interativo do agente. Algoritmo genético. Processo de aprendizagem.

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## APPENDIX A

### TABLE 1 – SAMPLE STATISTICS OF RETURNS.

| | Real | Sim. 1 | Sim. 2 | Sim. 3 | Sim. 4 | Sim. 5 |
|---|---|---|---|---|---|---|
| Mean | 1.28e-3 | -3.32e-3 | -1.41e-3 | -1.17e-3 | -2.56e-3 | -1.19e-3 |
| Variance | 0.46 | 0.14 | 0.13 | 0.23 | 0.04 | 0.10 |
| Kurtosis | 0.69* | 1.11* | 6.21* | 1.06* | 3.21* | -0.44* |
| Skewness | -0.07 | 1.13* | 0.24* | 0.11 | 0.23* | -0.02 |
| BJ- test | 29.49* | 375.55* | 2272.92* | 69.87* | 618.34* | 11.68* |

* and ** indicates statistical significance at the conventional 1 % and 5 % levels.

Source – Produced by authors.

### TABLE 2 – AUTOCORRELATIONS OF RETURNS.

| | Real | Sim. 1 | Sim. 2 | Sim. 3 | Sim. 4 | Sim. 5 |
|---|---|---|---|---|---|---|
| *Row data* | | | | | | |
| Q(8) | 9.86 | 179.33* | 46.55* | 143.31* | 146.10* | 159.80* |
| Q(12) | 14.45 | 182.30* | 50.78* | 145.22* | 211.88* | 184.79* |
| Q(16) | 19.18 | 182.80* | 56.46* | 155.46* | 254.97* | 191.02* |
| *Squared values* | | | | | | |
| Q(8) | 9.33 | 26.86* | 17.49** | 112.56* | 415.63* | 59.91* |
| Q(12) | 14.29 | 29.20* | 24.18** | 114.54* | 533.39* | 87.40* |
| Q(16) | 15.77 | 32.04* | 27.62** | 117.86* | 604.18* | 100.59* |

(continue)

TABLE 2 – AUTOCORRELATIONS OF RETURNS (continuation).

| | Real | Sim. 1 | Sim. 2 | Sim. 3 | Sim. 4 | Sim. 5 |
|---|---|---|---|---|---|---|
| *Absolute values* | | | | | | |
| Q(8) | 18.83 | 102.67* | 20.01** | 139.86* | 1176.84* | 277.47* |
| Q(12) | 21.18 | 109.33* | 30.07* | 142.54* | 1538.83* | 338.24* |
| Q(16) | 24.46 | 117.78* | 34.02* | 151.25* | 1788.69* | 367.35* |

* and ** indicates statistical significance at the conventional of the Ljung-Box statistic 1 % and 5 % levels.



Source – Produced by authors.

TABLE 3 – UNIT ROOT TESTS OF (LOG) FOREIGN EXCHANGE RATES.

| | Real | Sim. 1 | Sim. 2 | Sim. 3 | Sim. 4 | Sim. 5 |
|---|---|---|---|---|---|---|
| $t(r_{-1})/t$ | -2.11 | -3.96* | -1.17 | -1.55 | -1.09 | -1.59 |
| $x_m(1)$ | 23.41 | 32.61 | 18.58 | 21.29 | 20.49 | 24.02 |
| $x_t(1)$ | 16.33 | 15.19 | 17.19 | 16.87 | 17.30 | 16.70 |

$t(r_{-1})/t$ is the Phillips-Perron test for unit root with constant and trend. The critical value are -3.96, -3.31 and –3.12 at the 1%, 5% and 10% levels respectively. $x_m(1)$ and $x_t(1)$ are the KPSS tests for unit root for lag l = 1 with constant and trend respectively. The critical values are 0.739, 0.463 and 0.347 and 0.216, 0.143 and 0.119 respectively at the 1%, 5% and 10 % level.

Source – Produced by authors.

TABLE 4 – BDS TESTS FOR THE FOREIGN EXCHANGE RETURNS AND ENTROPY.

| s/e = 0,5 | m = 2 | m = 3 | m = 4 | m = 5 | m = 6 | m = 7 | m = 8 | m = 9 | m = 10 |
|---|---|---|---|---|---|---|---|---|---|
| Real Serie | -0.22 | -0.19 | -0.17 | -0.02 | 0.26 | 0.29 | 0.84 | 1.16 | 1.62 |
| Simulation 1 | 4.40 | 5.05 | 5.91 | 6.94 | 8.37 | 10.22 | 12.92 | 16.14 | 19.38 |
| Simulation 2 | 1.74 | 1.82 | 2.05 | 2.34 | 2.61 | 2.89 | 3.23 | 3.61 | 3.88 |
| Simulation 3 | 3.47 | 5.77 | 7.83 | 10.15 | 13.42 | 17.76 | 23.60 | 32.22 | 42.63 |
| Simulation 4 | 5.39 | 6.60 | 8.49 | 10.82 | 13.79 | 17.88 | 23.07 | 30.57 | 40.96 |
| Simulation 5 | 8.28 | 13.49 | 21.42 | 34.86 | 58.23 | 101.35 | 181.10 | 328.99 | 601.26 |

(continue)

TABLE 4 – BDS TESTS FOR THE FOREIGN EXCHANGE RETURNS AND ENTROPY (continuation).

| *s/e* = 1 | m = 2 | m = 3 | m = 4 | m = 5 | m = 6 | m = 7 | m = 8 | m = 9 | m = 10 |
|---|---|---|---|---|---|---|---|---|---|
| Real Serie | -0.13 | -0.08 | -0.06 | -0.01 | 0.11 | 0.26 | 0.44 | 0.68 | 0.88 |
| Simulation 1 | 3.20 | 3.30 | 3.63 | 4.01 | 4.61 | 5.35 | 6.35 | 7.44 | 8.46 |
| Simulation 2 | 1.15 | 1.18 | 1.29 | 1.46 | 1.63 | 1.80 | 2.00 | 2.20 | 2.35 |
| Simulation 3 | -1.41 | -0.21 | 0.26 | 0.53 | 0.79 | 1.03 | 1.20 | 1.49 | 1.71 |
| Simulation 4 | 3.85 | 4.74 | 6.02 | 7.50 | 9.15 | 11.13 | 13.48 | 16.89 | 21.18 |
| Simulation 5 | 4.33 | 6.84 | 9.75 | 13.83 | 19.60 | 27.98 | 40.45 | 59.76 | 88.68 |

| *s/e* = 1,5 | m = 2 | m = 3 | m = 4 | m = 5 | m = 6 | m = 7 | m = 8 | m = 9 | m = 10 |
|---|---|---|---|---|---|---|---|---|---|
| Real Serie | -0.07 | -0.03 | 0.00 | 0.04 | 0.09 | 0.17 | 0.26 | 0.35 | 0.43 |
| Simulation 1 | -0.46 | -0.47 | -0.40 | -0.37 | -0.33 | -0.28 | -0.23 | -0.15 | -0.06 |
| Simulation 2 | -0.23 | -0.20 | -0.13 | -0.10 | -0.11 | -0.13 | -0.13 | -0.14 | -0.17 |
| Simulation 3 | -0.96 | -0.49 | -0.38 | -0.29 | -0.23 | -0.16 | -0.11 | -0.05 | 0.00 |
| Simulation 4 | 10.33 | 10.92 | 12.87 | 15.20 | 18.20 | 21.81 | 26.27 | 31.96 | 39.02 |
| Simulation 5 | 3.21 | 4.67 | 6.02 | 7.63 | 9.55 | 11.92 | 14.93 | 19.00 | 24.15 |

| *s/e* = 2 | m = 2 | m = 3 | m = 4 | m = 5 | m = 6 | m = 7 | m = 8 | m = 9 | m = 10 |
|---|---|---|---|---|---|---|---|---|---|
| Real Serie | -0.02 | 0.01 | 0.04 | 0.06 | 0.09 | 0.12 | 0.16 | 0.20 | 0.23 |
| Simulation 1 | -0.55 | -0.56 | -0.47 | -0.44 | -0.41 | -0.36 | -0.32 | -0.25 | -0.17 |
| Simulation 2 | -0.37 | -0.31 | -0.26 | -0.23 | -0.23 | -0.26 | -0.26 | -0.27 | -0.32 |
| Simulation 3 | -0.08 | 0.04 | 0.07 | 0.09 | 0.09 | 0.09 | 0.09 | 0.10 | 0.11 |
| Simulation 4 | 0.43 | 0.54 | 0.73 | 0.92 | 1.08 | 1.23 | 1.37 | 1.52 | 1.66 |
| Simulation 5 | 0.14 | 0.26 | 0.37 | 0.50 | 0.61 | 0.71 | 0.81 | 0.90 | 0.99 |

| Entropy | |
|---|---|
| Real Serie | 0.68 |

(continue)

TABLE 4 – BDS TESTS FOR THE FOREIGN EXCHANGE RETURNS AND ENTROPY (continuation).

| | |
|---|---|
| Simulation 1 | 0.62 |
| Entropy | |
| Simulation 2 | 0.37 |
| Simulation 3 | 0.52 |
| Simulation 4 | 0.53 |
| Simulation 5 | 0.54 |



*s* is the standard deviation of the exchange rates, m refers to the embedding dimension, *e* is the distance parameter and is chosen to be the fraction of the standard deviation of the data. BDS test is distributed standard normal asymptotically. The entropy is the sum of the positive Lyapunov exponents and is computed for m = 5.

Source – Produced by authors.